Atomic and nuclear physics
Introductory experiments

Millikan experiment

Determining the electric unit charge after Millikan and verifying the charge quantification - Measuring with CASSY

Description from CASSY Lab 2

For loading examples and settings, please use the CASSY Lab 2 help.
Millikan’s experiment

can also be carried out with Pocket-CASSY

Experiment description
In 1910, R. A. Millikan succeeded in demonstrating the quantized occurrence of smallest amounts of electrical charge using his famous oil droplet method. He observed charged oil droplets in a vertical electrical field of a plate capacitor with the plate distance d and determined the charge q of a floating droplet from its radius r and the electrical field E=U/d. In his experiment, he found that q only occurs as integral multiples of an elementary charge e, i.e. q = n·e.

Theory
When an oil droplet with radius $r_0$ falls with the velocity $-v_1$, this droplet is subject to Stokes friction, which leads to the upward directed force $F_1 = 6\pi \eta r_0 v_1$ ($\eta$ = viscosity of air). When the oil droplet rises with the velocity $v_2$ in an external electrical field E, the downward directed force due to Stokes friction is $F_2 = -6\pi \eta r_0 v_2$. The difference between these two forces is exactly the force $q_0 E$ exerted by the applied electrical field E, i.e.

$$q_0 E = q_0 U/d = F_1 - F_2 = 6\pi \eta r_0 (v_1 + v_2) \quad \text{or} \quad q_0 = 6\pi \eta r_0 d (v_1 + v_2) / U.$$  

In order to determine the charge $q_0$, only the radius $r_0$ of the oil droplet under consideration is required, which, however, is easily obtained from the equilibrium of forces between its resultant gravitational force $F = -V \Delta \rho g$ and the Stokes frictional force $F_1$ in the case of the falling droplet, where $\Delta \rho$ is the difference between the densities of oil and air.
Thus we have:

\[ 0 = F + F_1 = \frac{-4}{3} \pi \cdot r_0^3 \cdot \Delta \rho \cdot g + 6 \pi \cdot \eta \cdot r_0 \cdot v_1 \quad \text{or} \]

\[ r_0 = \sqrt{\frac{(9 \eta v_1}{2 \Delta \rho g}).} \]

For a more precise determination of the charge \( q \), it has to be taken into account that Stokes friction has to be corrected for very small radii \( r \) because these are of the order of magnitude of the mean free path of the air molecules. The corrected formula for the frictional force, which depends on the air pressure \( p \), reads

\[ F = 6 \pi \eta r v / (1 + b/r \cdot p) \]

with \( b = 80 \ \mu m \cdot hPa \) (constant).

With the abbreviation \( A = b/p \), the corrected radius \( r \) is

\[ r = \sqrt{(r_0^2 + A^2/4)} - A/2 \]

and the corrected charge \( q \) is

\[ q = q_0 / (1 + A/r)^{1.5}. \]

**Floating method**

In this version of the experiment, the voltage \( U \) at the plate capacitor is adjusted such that a particular oil droplet floats, i.e. the rising velocity \( v_2=0 \). The falling velocity \( v_1 \) is measured after switching off the voltage \( U \) at the capacitor. Because of \( v_2=0 \), the above formulas are slightly simplified.

However, \( v_2=0 \) cannot be adjusted very precisely for fundamental reasons. Therefore the floating method leads to larger measurement errors and broader scattering in the frequency distribution than in the case of the following method.

**Falling/rising method**

In the second version, the two velocities \( v_1 \) and \( v_2 \) the voltage \( U \) are measured. This method makes possible more precise measured values than the floating method because the velocity \( v_2 \) is really measured.

**Equipment list**

1. Sensor-CASSY 524 010 or 524 013
2. CASSY Lab 2 524 220
3. Timer box 524 034
4. Millikan apparatus 559 411
5. Millikan supply unit 559 421
6. Connecting lead, 50 cm, red 500 421
7. Pairs of cables, 50 cm, red and blue 501 45
8. Pair of cables, 50 cm, black 501 451
9. PC with Windows XP/Vista/7

**Experiment setup (see drawing)**

Set up the Millikan apparatus as described in the instruction sheet, fill in oil, and set up the circuit as shown in the drawing. Connect the stopwatch output 1 to the input E and the stopwatch output 2 to the input F of the timer box. Connect the voltage output of the supply unit to the input B of the Sensor-CASSY.

Attention: the microscope creates an inverted image. Therefore all directions of motions are inverted. In the following, however, the real motion is described.

For a better demonstration of the oil droplets, it is recommended to record the image of the microscope by means of a video camera (e.g. VideoFlex from ken-a-vision). In this case, the camera can record upside down so that the visible direction of motion corresponds to the real direction of motion.

**Carrying out the experiment**

a) Floating method

- **Load settings**
  - Orient the eyepiece micrometer vertically and turn the lens holder of the eyepiece until you can clearly see the micrometer scale.
  - First set the switches U and t in the down position.
• Switch the voltage at the capacitor on with the switch U and adjust it with the rotary potentiometer so (400-600 V) that a selected oil droplet rises with a velocity of approximately 1-2 scale graduation marks per second (i.e. it is seen falling when observed through the eyepiece). Then reduce the voltage until the oil droplet floats.
• Switch the voltage at the capacitor off with the switch U.
• As soon as the oil droplet is located at the height of a selected scale graduation mark, start the time measurement with the switch t.
• As soon as the oil droplet has fallen by another 20 scale graduation marks (corresponds to 1 mm), switch the voltage at the capacitor on again with the switch U.
• Enter the measured values of the falling time \( t_1 \) and the voltage \( V \) in the table with \( \mathbb{Q} \). The calculated charge \( q \) is entered in the histogram automatically.
• Repeat the measurement for other oil droplets.

b) Falling/rising method

Load settings
• Orient the eyepiece micrometer vertically and turn the lens holder of the eyepiece until you can clearly see the micrometer scale.
• First set the switches U and t in the down position.
• Switch the voltage at the capacitor on with the switch U and adjust it with the rotary potentiometer so (400-600 V) that a selected oil droplet rises with a velocity of approximately 1-2 scale graduation marks per second (i.e. it is seen falling when observed through the eyepiece).
• Switch the voltage at the capacitor off with the switch U.
• As soon as the oil droplet is located at the height of a selected scale graduation mark, start the time measurement with the switch t.
• As soon as the oil droplet has fallen (i.e. risen as observed in the eyepiece) by another 20 scale graduation marks (corresponds to 1 mm), switch the voltage at the capacitor on again with the switch U. Thereby the time measurement \( t_2 \) is started automatically.
• As soon as the oil droplet is at the height of the first scale graduation mark again, stop the time measurement with the switch t.
• Enter the measured values of the falling time \( t_1 \), the rising time \( t_2 \) and the voltage \( V \) in the table with \( \mathbb{Q} \). The calculated charge \( q \) is entered in the histogram automatically.
• Repeat the measurement for other oil droplets.

Evaluation
In the evaluation, mean values can be drawn from measured frequency distribution and the relation \( q = n \cdot e \) (with \( e = 1.6022 \cdot 10^{-19} \) C) can be confirmed.

Remarks
If oil droplets with a small charge are selected, statistical significance is achieved more quickly. Oil droplets carrying a small charge are identified by their small size and their slow motion in the electric field.
If there is not enough time during the lessons for observing about 20-30 oil droplets, the example with its measured values can be loaded instead of merely the settings. The new measured values then appear in the histogram as red bars thus confirming the results of the example drawn in black within the usual statistical uncertainty.
In order to measure negative charges \( q \), the connections at the plate capacitor and at the input B of the CASSY have to be exchanged.
If the local air pressure differs considerably from 1013 hPa, the air pressure should be changed correspondingly in the formula defining the correction parameter \( A \). However, the displayed example values may then be wrong.