**Optics**

**Spectrometer**

**Grating spectrometer**

---

**Determination of the grating constants of the holographic grating using a He-Ne-laser**

---

**Objects of the experiment**

- Determination of the grating constant in the Littrow setup.
- Determination of the grating constant for various angles.

---

**Principles**

In the experiment the lattice constant $g$ of the holographic grating is determined by means of a He-Ne laser for which the wavelength is well known.

The interference arises in reflection:

$$
\lambda = a + b = \Delta
$$

With an angle of incidence $\alpha$ and the diffraction angle $\beta$ ($\frac{a}{g} = \sin \alpha$ and $\frac{b}{g} = \sin \beta$) the result for the grating constant $g$ is:

$$
g = \frac{\lambda}{\sin \alpha + \sin \beta} \quad (1)
$$

The resulting number of grating lines $N$ per mm is:

$$
N = \frac{1}{1000 \cdot g} \quad (2)
$$

The angles $\alpha$ and $\beta$ can be determined for this setup from the angle of the holographic grating with respect to the optical axis $\omega_{\text{pointer}}$ (angle of rotation) and the angle between the optical benches $\omega_{\text{benches}}$ (direction of the diffracted light beam). It can be observed for three different situations:

1. The angle of incidence is larger than the angle of diffraction: $\alpha > \beta$

   The angle of incidence $\alpha$ corresponds to the angle of rotation of the pillar with the holographic grating:

   $$
   \alpha = \omega_{\text{pointer}} \quad (3)
   $$

   The result for the angle of diffraction $\beta$ is (see fig. 2):

   $$
   \beta = \omega_{\text{pointer}} + \omega_{\text{benches}} - 180^\circ \quad (4)
   $$

   The limiting angle of observation in this situation is $\alpha = 90^\circ$.

   The result for the angle of diffraction from (1) is:

   $$
   \sin \beta = \frac{\lambda}{g} - 1 \quad (5)
   $$
2. The angle of incidence is equal to the angle of diffraction:
\[ \alpha = \beta \]

In this the so-called Littrow setup, the 1st order maximum is reflected in the direction of the incident light beam.

With equation (1) the result is
\[ g = \frac{\lambda}{2 \cdot \sin \alpha} \] (6)
i.e. with \( \alpha = \omega_{\text{pointer}} \) (3):
\[ g = \frac{\lambda}{2 \cdot \sin \omega_{\text{pointer}}} \] (7)

The advantage of the Littrow setup is that the measuring error for \( g \) (or \( \lambda \)) is small because only one angle needs to be determined. The measuring error can be further reduced by calculating the average value of the angles for both sides. They are determined by rotating the holographic grating clockwise and counter-clockwise.

3. The angle of incidence is smaller than the diffraction angle:
\[ \alpha < \beta \]

The result for the angle of diffraction \( \beta \) is (see fig. 3):
\[ \beta = 180^\circ + \omega_{\text{pointer}} - \omega_{\text{benches}} \] (8)

Fig. 3: For calculating the angle for \( \alpha < \beta \)

The limiting angle of observation in this situation is \( \beta = 90^\circ \). The result for the angle of diffraction from (1) is:
\[ \sin \alpha = \frac{\lambda}{g} - 1 \] (9)

### Apparatus

- 1 optical bench with standard profile, 1 m ..........460 32
- 1 optical bench with standard profile, 0.5 m ..........460 335
- 1 rail connector with circular scale .......................460 341
- 5 optics riders 90/50 .....................................460 374
- 1 He-Ne-laser .............................................471 830
- 1 lens in frame, \( f = 5 \text{ mm} \) ..........................460 01
- 1 projection objective ....................................460 13
- 1 holographic grating, 2400 lines / \text{mm} ..........471 27
- 1 lens in frame, \( f = 300 \text{ mm} \) ..........................460 09
- 1 screen ................................................................441 531

### Safety notes

The He-Ne-laser conforms to the “Safety technology requirements for teaching and training equipment DIN 58126 part 6 for lasers class 2”. Provided the corresponding information in the operating instructions is observed, experimentation with the He-Ne-laser is safe.

- Do not look into the direct or the reflected laser beam.
- Avoid exceeding the dazzling limit (i.e. no observer may feel dazzled).
**Setup**

Fig. 4: Experimental setup

- First of all align the two optical benches with their feet at the same level and fix them using the adjustment screws.
- Screw the two rails together by means of the rail connector so that the circular angle scale is firmly attached to the rail on which the laser is mounted.
- Set up the optical elements as shown in the figure and adjust to the same height.
- Mount the laser approx. at the centre of the 1 m long optical bench and the holographic grating (in the rail connector) in such a way that the laser beam hits the centre of the holographic grating.
- Set the pointer on the pillar of the rotary joint to precisely 0° and rotate the holographic grating counter-clockwise (angle +) until the 1st order maximum is reflected in the direction of the incident light beam. To do this observe the reflection on the front of the laser.
- Measure the angle of rotation $\omega_{\text{pointer}}$ of the holographic grating.
- Then rotate the holographic grating clockwise (angle -) and repeat the measurement.

**Carrying out the experiment**

a) Determination of the grating constant in the Littrow setup: $\alpha = \beta$

- Rotate the holographic grating counter-clockwise (angle +) until the 1st order maximum is reflected in the direction of the incident light beam. To do this observe the reflection on the front of the laser.
- Measure the angle of rotation $\omega_{\text{pointer}}$ of the holographic grating.
- Then rotate the holographic grating clockwise (angle -) and repeat the measurement.

b) Determination of the grating constant for $\alpha > \beta$

- First of all determine from the limiting angle the angle between the rails $\omega_{\text{benches}}$ at which the 1st order maximum will be observed with the smallest diffraction angle $\beta$.
- Rotate the optical bench 0.5 m clockwise until $\omega_{\text{benches}} = 125^\circ$.
- Rotate the pillar with the holographic grating until the 1st order maximum hits the lens (f = 300 mm). If necessary, shift the lens until the screen is precisely located at its focal point.
- If necessary, slightly correct the angle $\omega_{\text{pointer}}$ of the pillar with the holographic grating until the 1st order maximum precisely falls on the centre of the screen.
- Read of the angle $\omega_{\text{benches}}$ between the optical benches and the angle $\omega_{\text{pointer}}$ of the holographic grating.
- Set $\omega_{\text{benches}}$ to a larger angle and repeat the experiment.

c) Determination of the grating constant for $\alpha < \beta$

- First of all determine from the limiting angle from (5) the angle between the rails $\omega_{\text{benches}}$ at which the 1st order maximum can be observed with the largest diffraction angle $\beta$.  
- Rotate the optical bench 0.5 m clockwise until $\omega_{\text{benches}} = 155^\circ$.
- Repeat the experiment as described under b).
- Set $\omega_{\text{benches}}$ to smaller angles and repeat the experiment.

Note: If there is not sufficient space the optical bench can be set up as described under b). In this case the holographic grating must correspondingly be rotated clockwise (angle + to angle -).
Measuring examples

a) Determination of the grating constant in the Littrow setup:
- Calculation of the grating constant from the known wavelength of the He-Ne laser $\lambda = 632.8$ nm

<table>
<thead>
<tr>
<th>Angle + $\omega_{\text{pointer}}$</th>
<th>Angle - $\omega_{\text{pointer}}$</th>
<th>Average $\omega_{\text{pointer}}$</th>
<th>Grating constant from (7) $g$</th>
<th>Lines per mm from (2) $N$</th>
</tr>
</thead>
<tbody>
<tr>
<td>49°</td>
<td>50°</td>
<td>49.5°</td>
<td>4.16 $10^{-7}$ m</td>
<td>2403</td>
</tr>
</tbody>
</table>

The average value found corresponds well to the specification of $N = 2400$ lines per mm.

b) Determination of the grating constant $\alpha > \beta$
- With equation (5) the limiting angle is: $\beta \approx 31^\circ$
  from this with (4) $\omega_{\text{benches}} \approx -59^\circ$ or (see fig. 2)
  $\omega_{\text{benches}} \approx 121^\circ$

<table>
<thead>
<tr>
<th>Angle between the optical benches $\omega_{\text{benches}}$</th>
<th>Angle of the holographic grating $\omega_{\text{pointer}}$</th>
<th>Diffraction angle (from 4) $\beta$</th>
<th>Lines per mm (from 1, 2) $N$</th>
</tr>
</thead>
<tbody>
<tr>
<td>125°</td>
<td>87°</td>
<td>32°</td>
<td>2439</td>
</tr>
<tr>
<td>135°</td>
<td>78.5°</td>
<td>33.5°</td>
<td>2421</td>
</tr>
<tr>
<td>145°</td>
<td>71°</td>
<td>36°</td>
<td>2423</td>
</tr>
<tr>
<td>155°</td>
<td>64°</td>
<td>39°</td>
<td>2415</td>
</tr>
<tr>
<td>Average:</td>
<td></td>
<td></td>
<td>2425</td>
</tr>
</tbody>
</table>

c) Determination of the grating constant for $\alpha < \beta$
- With equation (9) the limiting angle is: $\alpha \approx 31^\circ$
  from this with (8) $\omega_{\text{benches}} \approx -59^\circ$ or (comp. fig. 3)
  $\omega_{\text{benches}} \approx 121^\circ$

<table>
<thead>
<tr>
<th>Angle between the optical benches $\omega_{\text{benches}}$</th>
<th>Angle of the holographic grating $\omega_{\text{pointer}}$</th>
<th>Diffraction angle (from 8) $\beta$</th>
<th>Lines per mm (from 1, 2) $N$</th>
</tr>
</thead>
<tbody>
<tr>
<td>155°</td>
<td>39°</td>
<td>64°</td>
<td>2415</td>
</tr>
<tr>
<td>145°</td>
<td>35.5°</td>
<td>70.5°</td>
<td>2403</td>
</tr>
<tr>
<td>135°</td>
<td>33°</td>
<td>78°</td>
<td>2406</td>
</tr>
<tr>
<td>125°</td>
<td>31.5°</td>
<td>86.5°</td>
<td>2403</td>
</tr>
<tr>
<td>Average:</td>
<td></td>
<td></td>
<td>2407</td>
</tr>
</tbody>
</table>