Optics

Wave optics
Diffraction

Diffraction
at a double slit
and at multiple slits

Objects of the experiments

■ Investigating diffraction at a double slit for various slit spacings.

■ Investigating diffraction at a double slit for various slit widths.

■ Investigating diffraction at multiple slits for various slit numbers.

Principles

The nature of light was a controversial issue for a long time. In 1690, Christiaan Huygens interpreted light as a wave phenomenon; in 1704, Isaac Newton described the light beam as a current of particles. This contradiction was resolved by quantum mechanics, and the idea of wave-particle duality came up.

Diffraction at a double slit:

A particularly clear indication of the wave character of light was provided by the experiment on diffraction at a double slit

by *T. Young*. Today this experiment can easily be reproduced with the intensive and coherent light of an He-Ne laser:

Due to diffraction of the parallel incoming light at two closely spaced slits of equal aperture, the light propagates also in the geometric shadow of the slit diaphragms (grey area in Fig. 1). Moreover, a system of bright and dark fringes is observed on a screen behind the double slit. This cannot be explained with the laws of geometrical optics.

An explanation is possible if wave properties are assigned to the light and if the slits are considered to be two coherent light sources whose light bundles superimpose. The superposition leads to destructive and constructive interference in certain directions. In a simple approach, the light bundles coming from the slits are first subdivided into (infinitely) many partial bundles. Then it can be made clear with the aid of Fig. 1 that maximum intensity occurs in directions in which there is exactly one partial bundle from the second slit which corresponds to any partial bundle from the first slit so that both interfere constructively. For light bundles

Fig. 1 Schematic illustration of diffraction at a double slit
b: slit width, d: slit spacing
L: distance between the screen and the double slit
x_2: distance of the second maximum from the centre
α_2: direction of observation for the second maximum
Δs_2: path difference of the principal rays
S: screen
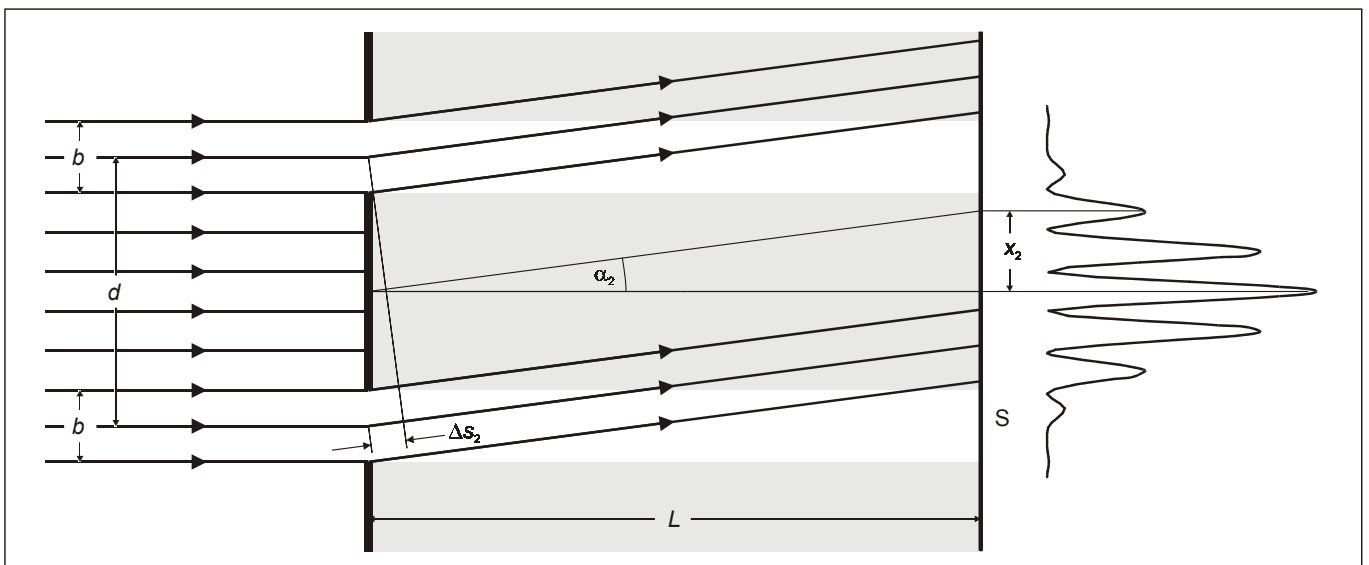
Apparatus

1 diaphragm with 3 double slits	469 84
1 diaphragm with 4 double slits	469 85
1 diaphragm with 5 multiple slits	469 86
1 He-Ne laser, linearly polarised	471 830
1 holder with spring clips	460 22
1 lens in frame, f = +5 mm	460 01
1 lens in frame, f = +50 mm	460 02
1 precision optical bench, 1 m	460 32
4 optics riders, H = 60 mm/B = 36 mm	460 370
1 translucent screen	441 53
1 saddle base	300 11

that emerge under the angle α_n this is true each time when the path difference Δs_n between the principal rays (drawn in the figure) is an integer multiple of the wavelength λ of the light:

$$\Delta s_n = n \cdot \lambda \qquad n = 0, \pm 1, \pm 2, \ldots \qquad (I)$$

For small diffraction angles the following relation holds approximately:

$$\frac{\Delta s_n}{d} \approx \alpha_n \approx \frac{x_n}{L} \qquad\qquad (II).$$

Hence, the intensity maxima are located on the screen at the positions (measured from the centre of the diffraction pattern)

$$x_n = n \cdot L \cdot \frac{\lambda}{d} \qquad n = 0, \pm 1, \pm 2, \ldots \qquad (III);$$

i.e. they are spaced at the distance

$$a = x_{n+1} - x_n = L \cdot \lambda \cdot \frac{1}{d} \qquad\qquad (IV).$$

Exactly in the middle between two intensity maxima there is an intensity minimum. Therefore the distance between a minimum and the next one is also given by Eq. (IV).

It should be mentioned explicitly that the present considerations are based on *Frauhofer*'s point of view, which means that parallel wave fronts of the light before and after the diffraction object are investigated. On the one hand this corresponds to a light source that is at an infinite distance

from the diffraction object, and on the other hand it corresponds to a screen that is at an infinite distance from the diffraction object. In the case of *Fresnel*'s point of view, the light source and the screen are at a finite distance from the diffraction object. However, the diffraction patterns are more easily calculated for *Fraunhofer* diffraction.

The intensity would be the same for all maxima, i.e. the bright fringes would exhibit the same brightness if the diffraction of light at the individual slits occurred with the same intensity in all directions. However, diffraction at a single slit depends on the angle of observation α. Therefore the diffraction pattern observed behind the double slit is modulated by diffraction at a single slit. For an exact calculation of the diffraction pattern, the oscillation states of all partial bundles that come from the slits are added up with their phase differences being taken into account. As a result the amplitude A of the field strength of the diffracted light is obtained at an arbitrary position x on the screen. From the amplitude distribution $A(x)$ calculated this way, the intensity distribution $I(x) = A^2(x)$ is derived immediately.

On the left of Fig. 2, the diffraction pattern of a double slit is shown for various slit spacings d with the same slit width b. It is clearly seen that the distance between the maxima decreases with increasing slit spacing. Their intensity is not constant because it is influenced by diffraction at single slits. Therefore it is sensible to determine the distance a on the screen defined in Eq. (IV) from the distance between the minima instead of the distance between the maxima.

On the right of the same figure, the diffraction pattern of a double slit is shown for various slit widths b with the same slit spacing d. The distance between the maxima is now the same in all three cases; however, the intensities are different because the influence of diffraction at single slits varies.

Diffraction at multiple slits:

The consideration regarding the determination of the maxima in diffraction at a double slit can be applied immediately to the diffraction at multiple slits with N equally spaced slits having the same aperture. If Eq. (I) is fulfilled, the light bundles of all N slits interfere constructively. Eqs. (III) and (IV) also hold for multiple slits.

Mathematically the determination of the intensity minima is more involved: a minimum between the n-th and the $(n+1)$-th maximum is found if the path difference between the principal rays of neighbouring slits fulfils the condition

$$\Delta s = n \cdot \lambda + m \cdot \frac{\lambda}{N} \quad m = 1, \ldots, N - 1 \qquad (V).$$

For this path difference, the partial bundles from the N slits interfere such that the total intensity is zero. This is illustrated in Fig. 3 with the aid of so-called pointer representations, in which the phase differences between the partial bundles coming from different slits are taken into account.

There are $N - 1$ minima between every pair of maxima. In between there are $N - 2$ so-called secondary maxima, whose intensity is weaker than that of the principal maxima. However, the latter is only true as long as the influence of diffraction at single slits can be neglected. In Fig. 4, the dependence of the diffraction pattern on the number of slits N is shown. As the slits are equally spaced, the distances between the principal maxima are equal for all slit numbers. With increasing slit number N, the intensity of the secondary maxima becomes weaker as compared with the principal maxima.

Safety notes

The He-Ne laser meets the "Safety Requirements for Teaching and Training Equipment - Laser, DIN 58126, Part 6" for lasers class 2. If the corresponding notes of the instruction sheet are observed, experimenting with the He-Ne laser is safe.

■ Never look into the direct or reflected laser beam.
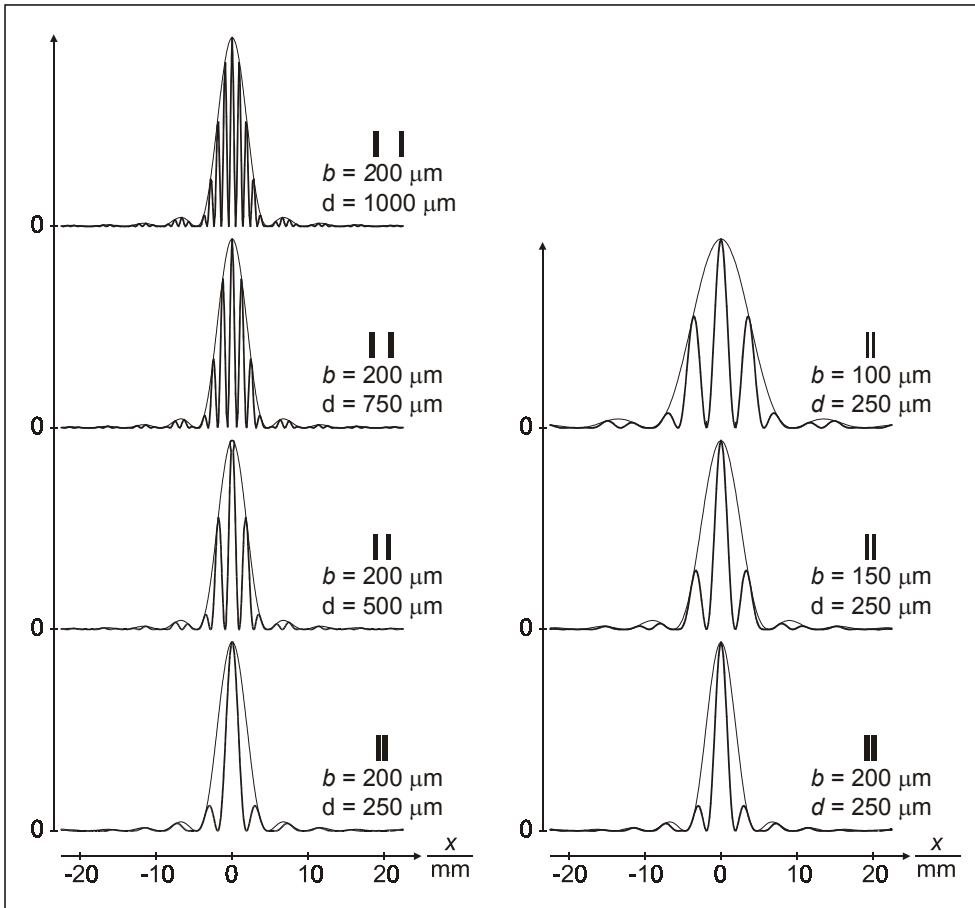
■ No observer must feel dazzled.

Fig. 2 Dependence of the diffraction pattern of a double slit on the slit spacing *d* (left) and the slit width *b* (right).

In each case, the diffraction pattern of a single slit with the corresponding slit width is drawn in the diagram with a smaller line width.

Fig. 3 Pointer representation of the addition of the oscillation amplitudes of *N* slits with the phase difference being taken into account.

If the path difference Δ*s* between two neighbouring slits is an integer multiple of λ, the maximum diffraction amplitude is obtained.

If the path difference Δ*s* is given by Eq. (V), the diffraction amplitude is zero.

Fig. 4 Dependence of the diffraction pattern of multiple slits on the slit number *N* for equal slit spacing *d* and equal slit width *b*.

The diffraction pattern of a single slit with equal slit width is drawn in the diagrams with a smaller line width.
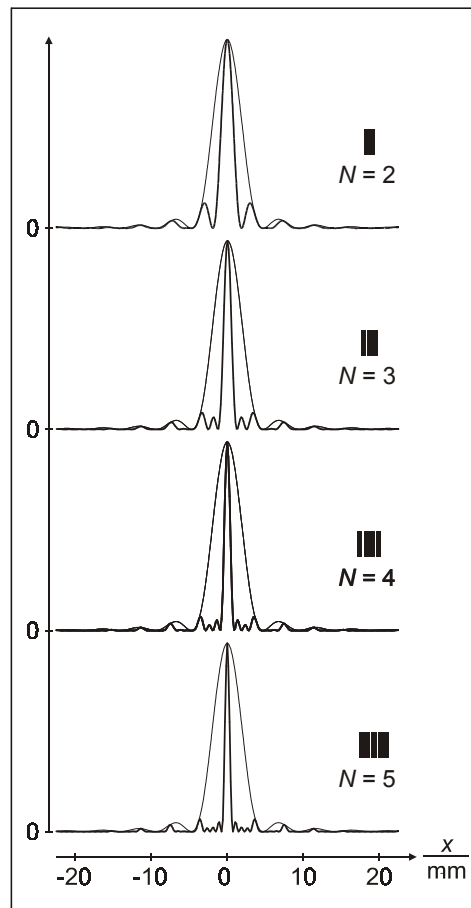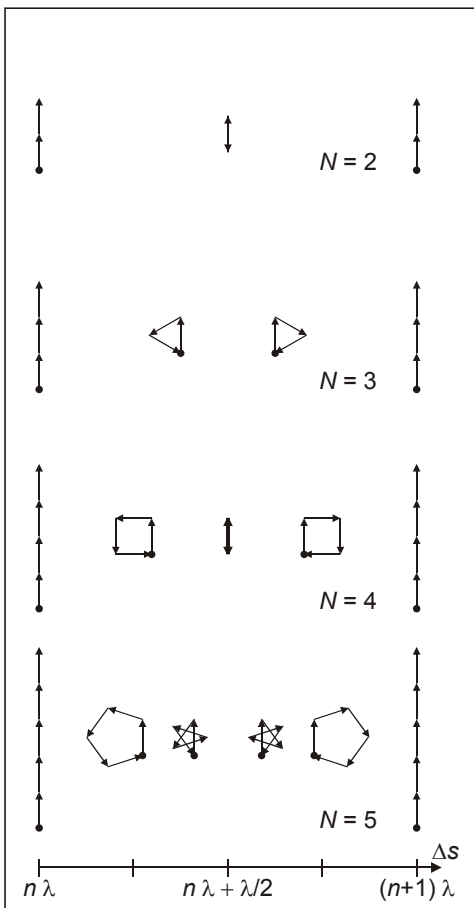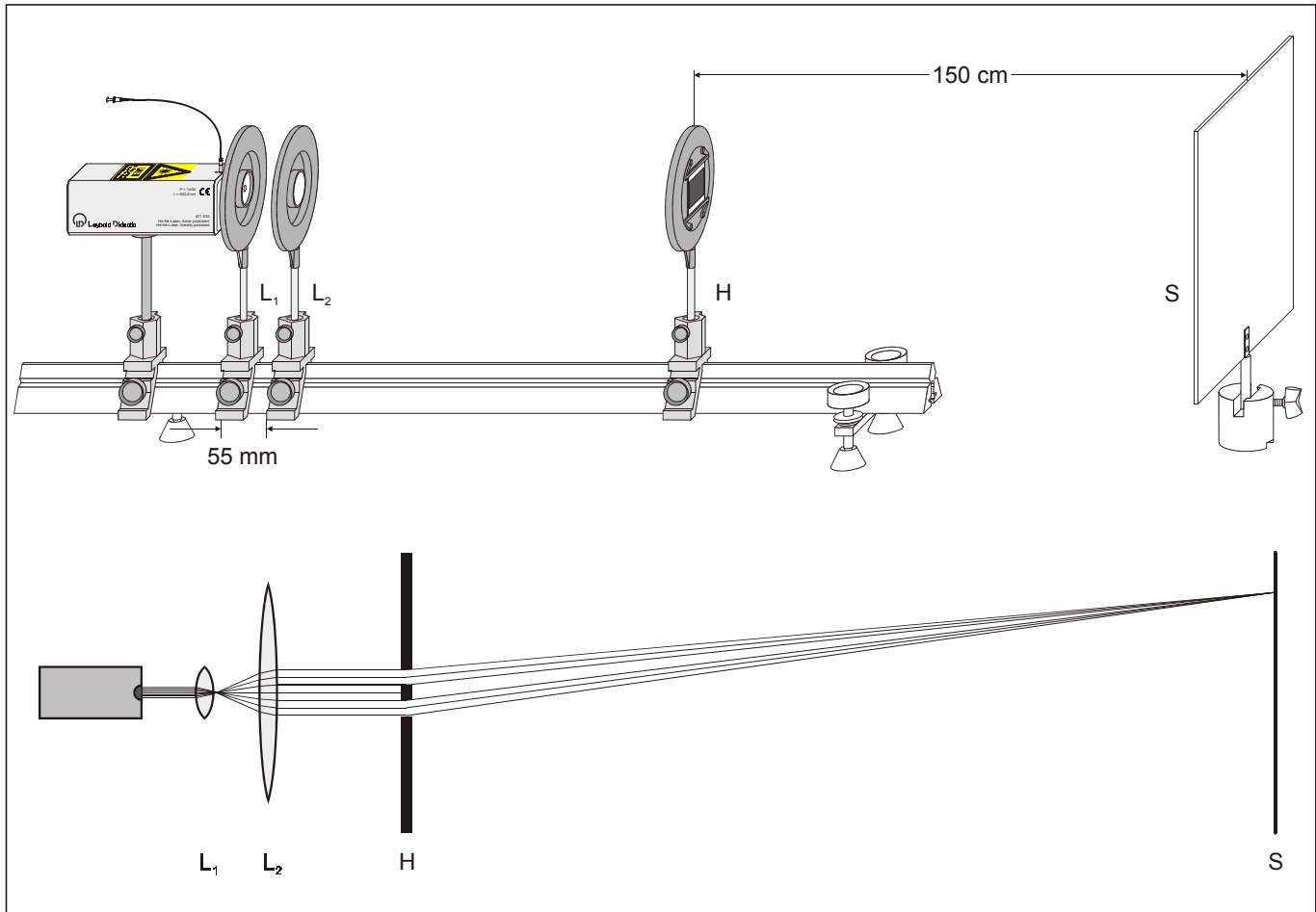
Fig. 5   Experimental setup (above) and schematic ray path (below)
for the observation of diffraction at a double slit and at
multiple slits.
L_1: lens f = +5 mm
L_2: lens f = +50 mm
H: holder for diffraction objects
S: screen

Setup

Remark: adjustments should be made in a slightly darkened room.

The total experimental setup is illustrated in Fig. 5. First the spherical lens L_1 with the focal length f = +5 mm expands the laser beam. The following converging lens L_2 with the focal length f = +50 mm is positioned so that its focus is located somewhat below the focus of the spherical lens. In this way the laser beam is slightly expanded and runs approximately parallel along the optical axis.

– Using an optics rider, attach the He-Ne laser to the optical bench as shown in Fig. 5.

– Set up the screen at a distance of approx. 1.90 m from the laser.

– Direct the laser towards the screen, and switch it on.

– Put the holder for diffraction objects H with the diaphragm with 4 double slits (469 85) on the optical bench at a distance of approx. 50 cm from the laser.

– Adjust the height of the laser so that the laser beam impinges on the centre of the diaphragm.

– Place the spherical lens L_1 with the focal length f = +5 mm at a distance of approx. 1 cm from the laser (the laser should illuminate the diaphragm evenly.)

– Remove the holder for diffraction objects H.

– Place the converging lens L_2 with the focal length f = +50 mm at a distance of approx. 55 mm behind the spherical lens L_1 and slide it along the optical bench towards the spherical lens L_1 until the image of the laser beam on the screen is sharp.

– Advance the converging lens L_2 on the optical bench somewhat further towards the spherical lens L_1 until the diameter of the laser beam on the screen has expanded to approx. 6 mm (the laser beam should now have a constant circular cross section along the optical axis).

– In order to check whether the beam diameter is constant between the lens and the screen, hold a sheet of paper in the ray path and observe the cross section of the beam along the optical axis.

– Put the holder for diffraction objects back into the ray path and shift it so that the distance between the screen and the diffraction object is 1.50 m.

– If necessary, slightly shift the lens L_2 until the diffraction pattern is sharp.

Carrying out the experiment

a) Dependence of diffraction at a double slit on the slit spacing *d*:

– Insert the diaphragm with 4 double slits (469 85) in the ray path, and observe the diffraction patterns of the double slits with the slit spacings *d* = 1.00 mm, 0.75 mm, 0.50 mm and 0.25 mm one after another.

– For each slit spacing draw conclusions on the influence of diffraction at a single slit from the intensities of the maxima.

– Hold a sheet of paper on the screen in each case, and mark the locations of the intensity minima (dark fringes!) within the central maximum of the single slit function with a soft pencil.

– Determine the (averaged) distance *a* between the intensity minima in each case.

b) Dependence of diffraction at a double slit on the slit width *b*:

– Insert the diaphragm with 3 double slits (469 84) in the ray path, and observe the diffraction patterns of the double slits with the slit widths *b* = 0.20 mm, 0.15 mm and 0.10 mm one after another.

– For each slit width draw conclusions on the influence of diffraction at a single slit from the intensities of the maxima.

– Hold a sheet of paper on the screen in each case, and mark the locations of the intensity minima (dark fringes!) within the central maximum of the single slit function with a soft pencil.

– Determine the (averaged) distance *a* between the intensity minima in each case.

c) Dependence of diffraction at multiple slits on the slit number *N*:

– Insert the diaphragm with 5 multiple slits (469 86) in the ray path, and observe the diffraction patterns of 2, 3, 4, 5 and 40 slits one after another.

– Identify the influence of diffraction at a single slit, the principal maxima and, for *N* = 3, 4 and 5, the secondary maxima.

– Hold a sheet of paper on the screen in each case, and mark the locations of the principal maxima (bright fringes!) with a soft pencil.

– Determine the (averaged) distance *a* between the intensity maxima in each case.

Measuring example

a) Dependence of diffraction at a double slit on the slit spacing *d*:

Table 1: distances *a* between the intensity minima for different slit spacings *d*

$\dfrac{d}{\text{mm}}$	$\dfrac{a}{\text{mm}}$
0.25	3.87
0.50	1.84
0.75	1.27
1.00	0.94

b) Dependence of diffraction at a double slit on the slit width *b*:

The distances between the maxima are equal for all slit widths. With decreasing slit width, the intensity is increasingly distributed to the maxima near the centre.

Table 2: distances *a* between the intensity minima for different slit widths *b*

$\dfrac{b}{\text{mm}}$	$\dfrac{a}{\text{mm}}$
0.10	3.75
0.15	3.36
0.20	3.67

c) Dependence of diffraction at multiple slits on the slit number *N*:

The distances between the principal maxima are equal for all slit numbers. The principal maxima themselves become narrower with increasing slit number *N*. For *N* = 3 to 5, there are N - 2 secondary maxima between two neighbouring principal maxima. The intensity of the secondary maxima becomes weaker with increasing *N*.

Table 3: distances *a* between the principal maxima for different slit numbers *N*

N	$\dfrac{a}{\text{mm}}$
2	3.81
3	3.78
4	3.81
5	3.73
40	3.79

Evaluation

a) Dependence of diffraction at a double slit on the slit spacing *d*:

Table 4: Distances *a* between the intensity minima and reciprocal slit distances d^{-1} (see Table 1)

$\dfrac{d^{-1}}{\text{mm}^{-1}}$	$\dfrac{a}{\text{mm}}$
4.00	3.87
2.00	1.84
1.33	1.27
1.00	0.94

In Fig. 6 the measured values from Table 1 are shown as a diagram. The distance *a* is plotted against the reciprocal of the slit spacing *d*. Within the accuracy of measurement, the measured values lie on the straight line through the origin drawn in the diagram. The slope of the straight line was calculated with the aid of Eq. (IV) from *L* = 150 cm and λ = 633 nm.

b) Dependence of diffraction at a double slit on the slit width *b*:

Mean value of the distances *a* from Table 2: *a* = 3.59 mm.

As λ = 633 nm and *L* = 150 cm are known, the slit spacing *d* can be calculated with the aid of the transformed Eq. (IV):

d = 0.26 μm

c) Dependence of diffraction at multiple slits on the slit number *N*:

Mean value of the distances *a* from Table 3: *a* = 3.78 mm.

As λ = 633 nm and *L* = 150 cm are known, the slit spacing *d* can be calculated with the aid of the transformed Eq. (IV):

d = 0.25 μm

Results

The diffraction pattern of diffraction at a double slit or at multiple slits, respectively, is determined by the slit spacing *d*, the slit number *N* and the slit width *b*.

The distance *a* between the principal maxima is inversely proportional to the slit spacing *d* and independent of *N* and *b*.

With increasing slit number *N*, the width of the principal maxima decreases because the number of minima (and of secondary maxima) increases.

The slit width *b* determines the influence of diffraction at a single slit on the diffraction pattern.

Fig. 6   Distance *a* between the intensity minima for diffraction at a double slit as a function the reciprocal of the slit spacing.