

## Generating Lissajous figures through electron deflection in crossed alternating magnetic fields

### Objects of the experiments

- Generating Lissajous figures through deflection of an electron beam in crossed alternating magnetic fields

### Principles

A Perrin tube can be used for investigating the properties of cathode rays. The existence of cathode rays, the straight-line movement in field-free space and the deflection in electric and magnetic fields was qualitatively investigated in experiments with the vacuum tube diode, the vacuum tube triode and the Maltese cross tube.

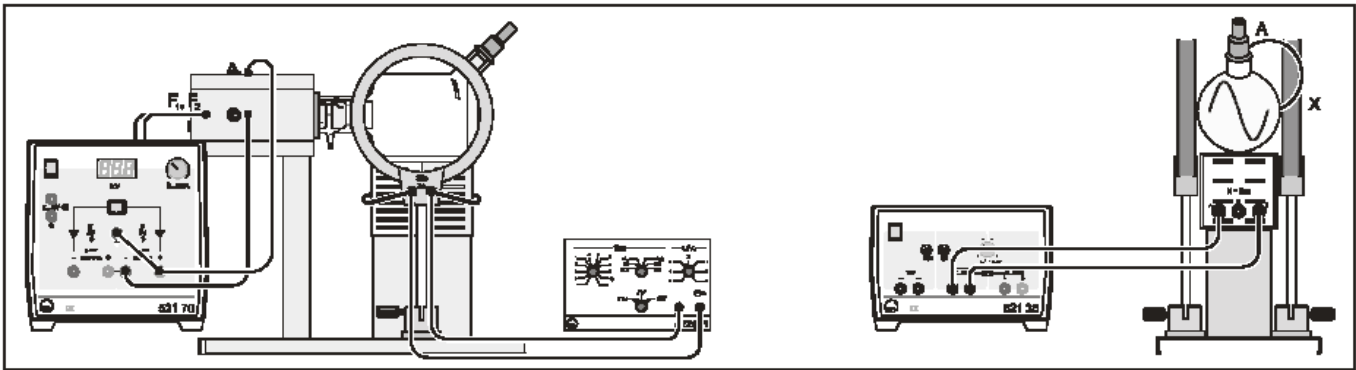
In the Perrin tube the polarity and the specific charge of the charge carriers can be determined (P3.8.4.1). On the fluorescent screen, in addition the deflection of the electron beam on account of electric and magnetic fields can be observed and investigated in more detail. The deflection in a magnetic field is caused by Lorentz force  $\vec{F} = q \cdot \vec{v} \times \vec{B}$  acting on the charge carrier and occurs perpendicularly both with respect to the direction of movement of the charge carrier and the field lines of the magnetic field.

In the experiment the deflection of electrons in a crossed magnetic alternating field is investigated. If an electron beam moves through two magnetic alternating fields whose field lines are perpendicular to each other the movements in the horizontal and vertical directions are superimposed. On the screen various figures become visible depending on the selected frequencies. If the ratio of the two applied frequencies is a rational number, closed and therefore static curves occur, the shape of which gives an indication of the frequency ratio and the relative phase of the applied alternating fields. Such curves are called Lissajous figures. For all other frequency ratios the curves are open and for this reason they appear to rotate. For the special case of equal frequencies and amplitudes and a phase shift of  $0^\circ$  (or  $180^\circ$ ) a diagonal line is obtained, for a phase shift of  $90^\circ$  (or  $270^\circ$ ) a circle. For differing amplitudes and a phase shift of  $0^\circ$  (or  $180^\circ$ ) again a single line is obtained, the slope of which depends on the amplitude ratio. For a phase shift of  $90^\circ$  (or  $270^\circ$ ) an ellipse is obtained whose semi-axes correspond to the deflection of the two applied alternating voltages. For any phase shift between these values a rotated ellipse is always obtained.



### Apparatus

1 Perrin tube .....	555 622
1 tube stand.....	555 600
1 Helmholtz pair of coils .....	555 604
1 coil with 500 windings[0] .....	562 14
1 variable low voltage transformer S .....	521 35
1 high voltage power supply 10 kV .....	521 70
1 function generator S 12, 0.1 Hz to 20 Hz.....	522 621
1 support block, set of 6[0].....	300 761
1 safety connection leads, 25 cm, red .....	500 611
2 safety connection leads, 50 cm, red .....	500 621
1 safety connection lead, 50 cm, blue .....	500 622
3 safety connection leads, 100 cm, red .....	500 641
3 safety connection leads, 100 cm, blue.....	500 642
2 safety connection leads, 100 cm, black .....	500 644

**Experimental setup:****Safety notes:**

The Perrin tube is a thin-walled evacuated glass cylinder. Danger of implosion!

- Do not expose the tube to any mechanical loads.
- Only connect the Perrin tube by means of safety connection cables.
- Observe the operating instructions for the Perrin tube (555 622) and the tube stand (555 600).

**Setup**

The experimental setup is shown in the figure. For setting up the steps described below are required:

- Carefully insert the Perrin tube into the tube stand.
- Connect sockets  $F_1$  and  $F_2$  on the tube stand for the cathode heater to the 10 kV output at the rear of the high voltage power supply.
- Connect socket C on the tube stand (cathode cap) to the negative pole and socket A (anode) to the positive pole of the 10 kV high voltage power supply and in addition earth the positive pole.
- Connect socket X (deflection plates) to socket A (anode).
- Set up the coil with 500 windings immediately beneath the Perrin tube by means of the support blocks and connect it to the output of the alternating voltage 0-20 V of the variable low voltage transformer S.
- Place the Helmholtz pair of coils in the positions marked with H (Helmholtz geometry) on the tube stand. Adjust the height of the coils in such a way that the centre of the coils is aligned with the beam axis. Connect the coil in series to the function generator so that the current flows through the coils in the same direction.

**Carrying out the experiment**

- Switch on the high voltage power supply and select an anode voltage between 2.5 and 5 kV. A green luminous spot will appear on the fluorescent screen.
- For generating a horizontal magnetic alternating field switch on the function generator and set an output voltage of approx. 2.5 V with a frequency  $f_1$  of approx. 1 Hz.
- Observe the luminous spot on the screen.
- Increase the frequency  $f_1$  to approx. 50 Hz and keep observing the image on the fluorescent screen. Then turn down the voltage to 0 V.
- For generating a vertical alternating field ( $f_2 = 50$  Hz) switch on the variable low voltage transformer and slowly increase the voltage to approx. 6 V.

- Without changing the frequency  $f_1 = 50$  Hz increase the voltage at the function generator to approx. 2.5 V. Observe the image on the fluorescent screen.
- Carefully vary the frequency  $f_1$  around 50 Hz.
- Then set markedly smaller or larger frequencies  $f_1$ .

**Observation**

If an alternating voltage with a frequency  $f_1$  of 1 Hz is applied the luminous spot will move from right to left on the screen. If the voltage is slowly increased the deflection of the luminous spot will increase both at the top and at the bottom. After increasing the frequency to 50 Hz a vertical line will be visible.

If an alternating voltage with a frequency of  $f_2 = 50$  Hz is connected to the low voltage coil, a horizontal line becomes visible on the fluorescent screen. If the voltage is increased the horizontal line lengthens both to the left and to the right.

If an alternating voltage with a frequency 50 Hz is connected to both the low voltage coil and the Helmholtz coils an ellipse appears on the fluorescent screen, which appears to rotate. For small modifications of the frequency  $f_1$  around 50 Hz the speed of rotation of the ellipse changes or the direction of rotation is reversed.

For larger frequency changes different figures appear. If the ratio of  $f_1$  and  $f_2$  is rational, the figures appear to be stationary.

**Evaluation**

The size of the deflection depends on the strength of the magnetic field and therefore on the current flowing through the coil. An increase of the applied voltage will lead to a higher current and therefore to a stronger deflection of the electron beam.

Up to a frequency of  $f_1 \approx 30$  Hz the movement of the spot can still be perceived, but above this it only appears to be a line because the eye can no longer resolve the movement of the spot. This effect is reinforced by the afterglow of the screen.

If an electron beam moves through two alternating magnetic fields whose field lines are perpendicular to each other, the movements in the horizontal and vertical directions are superimposed. On the screen various figures become visible depending on the selected frequencies. Such curves are called Lissajous figures.

The horizontal and vertical extent of the Lissajous figures on the fluorescent screen depend on the size of the applied alternating voltages. If the ratio of  $f_1$  and  $f_2$  is rational, closed and therefore stationary figures are obtained. In the figures

on the right the curves are shown schematically for various frequency ratios and phase shifts.

If the frequencies of the two fields are equal the Lissajous figure is, depending on the relative phase shift of the oscillations, a line, a circle or an ellipse. A rotating ellipse indicates that the applied frequencies are not exactly equal. If the frequency ratio is changed different figures are obtained.

The frequency ratio can be determined from the number of maxima along the outer edges which are found for a single runthrough. For a frequency ratio of 3:2 it can be seen that the curve has 3 maxima on top and bottom, but on the sides 2 maxima. A special case is presented when a maximum degenerates into a line. In this case several maxima are located at the same place. They have to, however, still be counted separately. In the example of the frequency ratio 4:3 with a phase shift of  $270^\circ$  the two bottom maxima degenerate into lines, the other maxima are traced double. For this reason when the complete sequence is counted a ratio of 4:3 is still obtained.

**Note:**

With the help of the Lissajous figures any alternating voltages can be analysed with regard to strength and frequency when on the fluorescent screen on the tube there is a scale and an alternating current of known strength and frequency is connected to the low voltage coil. This allows the setup used here to be used as a model for an oscilloscope.

