 Objects of the experiments
- Generating a standing microwave through reflection from a metal plate.
- Measuring the field distribution in the standing wave.
- Determining the wavelength $\lambda_0$ from the distance between neighbouring nodes.
- Demonstrating the shortening of the wavelength when a dielectric is introduced.
- Determining the refractive index $n$ of the medium.

Principles

If an incoming microwave impinges on a metal plate whose surface is perpendicular to the direction of propagation of the wave, a wave that propagates in the opposite direction arises due to reflection. The reflected wave superimposes upon the incoming wave, thus giving rise to a standing wave. For describing the wave mathematically, usually the electric field $E$ of the microwave field is considered:

For instance, a propagating wave with the frequency $f$ and the wavelength $\lambda_0$ has the form

$$E_1(x,t) = E_0 \cdot \cos \left( \frac{2\pi}{\lambda_0} \cdot f \cdot t - \frac{2\pi}{\lambda_0} \cdot x \right)$$  \hspace{1cm} (I).

An ideal reflection at $x = 0$ gives rise to the standing wave

$$E(x,t) = 2 \cdot E_0 \cdot \sin \left( \frac{2\pi}{\lambda_0} \cdot x \right) \cdot \sin \left( 2\pi \cdot f \cdot t \right)$$  \hspace{1cm} (II).

The shape of its amplitude

$$E_A(x) = 2 \cdot E_0 \cdot \sin \left( \frac{2\pi}{\lambda_0} \cdot x \right)$$  \hspace{1cm} (III)

is stationary (see Fig. 1); i.e. there are locations $x_i$ where the amplitude is zero at all times or where it has a maximum at all times. In communications engineering, this characteristic is called standing wave ratio. From the shape of the standing wave ratio, the wavelength $\lambda$ can be determined, as the distance between two antinodes (maxima) or two nodes (minima) corresponds to half a wavelength. In general, nodes are easier to measure. Therefore the distance between them is usually measured.

The wavelength depends on the medium in which the wave propagates. In a dielectric with the refractive index $n$, the wavelength $\lambda_n$ is shortened if compared with the wavelength in vacuum $\lambda_0$ (or the wavelength in air, which, to a good approximation, is equal to that in vacuum):

$$\lambda_n = \frac{\lambda_0}{n}$$  \hspace{1cm} (IV).
### Apparatus

<table>
<thead>
<tr>
<th>Item</th>
<th>Code</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 Gunn power supply with amplifier</td>
<td>737 020</td>
</tr>
<tr>
<td>1 Gunn oscillator</td>
<td>737 01</td>
</tr>
<tr>
<td>1 large horn antenna</td>
<td>737 21</td>
</tr>
<tr>
<td>1 stand rod, 245 mm, with thread</td>
<td>309 06 578</td>
</tr>
<tr>
<td>1 E-field probe</td>
<td>737 35</td>
</tr>
<tr>
<td>1 physics microwave accessories I</td>
<td>737 27</td>
</tr>
<tr>
<td>1 physics microwave accessories II</td>
<td>737 275</td>
</tr>
<tr>
<td>1 voltometer, DC, U ≤ 10 V</td>
<td>e.g. 531 100</td>
</tr>
<tr>
<td>3 saddle bases</td>
<td>300 11</td>
</tr>
<tr>
<td>2 BNC cables, 2 m</td>
<td>501 022</td>
</tr>
<tr>
<td>1 pair of cables, 100 cm, black</td>
<td>501 461</td>
</tr>
<tr>
<td>additionally recommended: 1 set of microwave absorbers</td>
<td>737 390</td>
</tr>
<tr>
<td>additionally recommended: 1 ruler</td>
<td></td>
</tr>
</tbody>
</table>

The shortening can be demonstrated from the shape of the standing wave ratio, as the distance between the first and the k-th node is

\[ x_{(D)} = (k - 1) \cdot \frac{1}{2} \cdot \lambda_0 \]  
\[ x_{(n)} = (k - 1) \cdot \frac{1}{2} \cdot \frac{\lambda_0}{n} \]

in air and in the dielectric. If the dielectric does not completely occupy the path of ray, i.e., if the sample of material has a smaller thickness \( D \), calculating the distance between the nodes is more complicated:

We have

\[ (x_{(D)} - D) + n \cdot D = (k - 1) \cdot \frac{\lambda_0}{2} \]  
\[ x_{(D)} = x_{(0)} - (n - 1) \cdot D \]

In order to reach the same node of the field strength, the probe has thus to be displaced by the path length \( d = (n - 1) \cdot D \)

towards the sample of the material (see Fig. 2). The refractive index in this case is

\[ n = \frac{D + d}{D} \]  
\[ D \cdot (n - 1) < \frac{\lambda}{2} \]

has to be fulfilled.

### Setup

**Remarks:**

Measuring results may be distorted by reflection of the microwaves from vertical surfaces of objects close to the experimental setup:

Choose the direction of transmission of the horn antenna so that reflecting surfaces are at a distance of at least 4 m. If possible, use microwave absorbers to build up a reflection-free measuring chamber.

If several experiments with microwaves are run at the same time, neighbouring Gunn oscillators can interfere:

Try to find a suitable arrangement of the experiments. In this case, use of microwave absorbers is mandatory to set up separate reflection-free measuring chambers.

The varying magnetic field of microwaves can induce voltages in cable loops:

Avoid cable loops.

The experimental setup is illustrated in Fig. 3.

- Attach the Gunn oscillator to the horn antenna with the quick connectors (b).
- Align the horn antenna horizontally, screw the 245 mm long stand rod into the corresponding thread and clamp it in a saddle base.
- Connect the Gunn oscillator to the output OUT via a BNC lead. Connect the E-field probe to the amplifier input and the voltmeter to the output DC OUT of the Gunn power supply.
- Set up the E-field probe in front of the centre of the horn antenna.
- Set the modulation frequency with the frequency adjuster (a) so that the multimeter displays maximum received signal.
- Set up the metal plate from the physics microwave accessories I at a distance of approx. 200 mm from the horn antenna.
- Align the metal plate, the E-field probe, and the horn antenna appropriately.

### Carrying out the experiment

**a) Field distribution in a standing wave:**

- Measure the received signal \( U \) of the E-field probe from \( x = -50 \text{ mm} \) to \(-150 \text{ mm}\) in steps of \( 5 \text{ mm}\) and take the measured values down (the position of the metal plate corresponds to \( x = 0 \text{ mm}\)).
- In addition, measure the locations and received voltages of at least five consecutive maxima and minima and take them down.

**b) Determining an unknown refractive index \( n \):**

- Increase the distance between the horn antenna and the metal plate.
- Use the E-field probe to find a minimum of the received voltage and take its position \( x \) down.
- Introduce the dielectric plate (PVC, 20 mm) from the physics microwave accessories II into the path of ray using the plate holder and a saddle base.
- Displace the E-field probe towards the plate and look for the new position \( x' \) of the minimum.

### Safety notes

Attention, microwave power! The microwave power released from the Gunn oscillator is approx. 10–15 mW, which is not dangerous to the experimenter. However, in order that students are prepared for handling microwave systems with higher power, they should practice certain safety rules.

- Never look directly into the transmitting horn antenna.
- Before positioning anything in the experimental setup, always disconnect the Gunn oscillator.
Measuring example

a) Field distribution in a standing wave:

Table 1: Received signal $U$ of the E-field probe in a standing wave

<table>
<thead>
<tr>
<th>$x$ mm</th>
<th>$U$ mV</th>
</tr>
</thead>
<tbody>
<tr>
<td>−50</td>
<td>1000</td>
</tr>
<tr>
<td>−55</td>
<td>2600</td>
</tr>
<tr>
<td>−60</td>
<td>4900</td>
</tr>
<tr>
<td>−65</td>
<td>2400</td>
</tr>
<tr>
<td>−70</td>
<td>1400</td>
</tr>
<tr>
<td>−75</td>
<td>5250</td>
</tr>
<tr>
<td>−80</td>
<td>3850</td>
</tr>
<tr>
<td>−85</td>
<td>560</td>
</tr>
<tr>
<td>−90</td>
<td>4900</td>
</tr>
<tr>
<td>−95</td>
<td>5250</td>
</tr>
<tr>
<td>−100</td>
<td>1200</td>
</tr>
<tr>
<td>−105</td>
<td>4200</td>
</tr>
<tr>
<td>−110</td>
<td>6100</td>
</tr>
<tr>
<td>−115</td>
<td>2800</td>
</tr>
<tr>
<td>−120</td>
<td>3550</td>
</tr>
<tr>
<td>−125</td>
<td>7000</td>
</tr>
<tr>
<td>−130</td>
<td>5200</td>
</tr>
<tr>
<td>−135</td>
<td>2450</td>
</tr>
<tr>
<td>−140</td>
<td>6000</td>
</tr>
<tr>
<td>−145</td>
<td>6300</td>
</tr>
<tr>
<td>−150</td>
<td>2850</td>
</tr>
</tbody>
</table>

b) Determining the refractive index $n$ of a PVC plate:

dielectric: PVC, $D = 20$ mm

Position of the field strength node without plate: $x = −134$ mm

Position of the field strength node after the plate has been introduced: $x′ = −123$ mm
**Evaluation and results**

**a) Field distribution in a standing wave:**

![Plot of the standing wave ratio](image)

Fig. 4 Plot of the standing wave ratio

Fig. 4 shows a plot of the standing wave ratio. In the plot, the measured values from Tables 1 and 2 and a calculated curve are drawn. In the calculation, it has been taken into account that the received signal $U$ of the E-field probe is proportional to the square of the electric field strength. As the amplitude of the wave transmitted by the microwave source decreases with increasing distance from the source, the amplitude of the standing wave decreases, too. The signal reflected at the wall (position $x = 0$) is weaker than the incoming signal. Therefore the minima do not go to zero.

Distance between the first and the seventh minimum:

$x_{(0)} = 151 \text{ mm} - 52 \text{ mm} = 99 \text{ mm}$.

From this we obtain the wavelength

$\lambda = 2 \cdot \frac{x_{(0)}}{6} = 33 \text{ mm}$

and the frequency

$f = \frac{c}{\lambda} = 9.1 \text{ GHz}$

($c = 3 \cdot 10^8 \text{ m/s}$).

For comparison:

For the dominant mode of the Gunn oscillator, the resonance frequency is given by

$f = \frac{c}{2 \cdot \sqrt{\frac{1}{s^2} + \frac{1}{b^2}}} = 9.4 \text{ GHz}$

$b = 23 \text{ mm}: \text{ cavity width}, s = 22 \text{ mm}: \text{ distance between the pinhole diaphragm and the Gunn element}$

**b) Determining the refractive index $n$ of a PVC plate:**

Displacement of the field strength node towards the PVC plate:

$d = x' - x = 13 \text{ mm}$

With the aid of Eq. (IX) the refractive index of the PVC plate is calculated:

$n = \frac{D + d}{D} = 1.65$