Determining the impedance in circuits with capacitors and ohmic resistors

Objects of the experiment
- Determining the total impedance and the phase shift in a series connection of a capacitor and a resistor.
- Determining the total impedance and the phase shift in a parallel connection of a capacitor and a resistor.

Principles

If an alternating voltage
\[ U = U_0 \cdot \cos(\omega \cdot t) \quad \text{with} \quad \omega = 2\pi \cdot f \quad (I) \]
is applied to a capacitor with the capacitance \( C \), the current flowing through the capacitor is
\[ I = U_0 \cdot \frac{1}{\omega \cdot C} \cdot \cos\left(\omega \cdot t + \frac{\pi}{2}\right) \quad (II). \]
Therefore a capacitive reactance
\[ X_C = \frac{1}{\omega \cdot C} \quad (III) \]
is assigned to the capacitor, and the current is said to be phase-shifted with respect to the voltage by 90° (see Fig. 1). The phase shift is often represented in a vector diagram.

Series connection

If the capacitor in connected in series with an ohmic resistor, the same current flows through both components. This current can be written in the form
\[ I = I_0 \cdot \cos(\omega \cdot t + \phi_s) \quad (IV) \]
where \( \phi_s \) is unknown for the time being. Correspondingly, the voltage drop is
\[ U_R = R \cdot I_0 \cdot \cos(\omega \cdot t + \phi_s) \quad (V) \]
at the resistor and
\[ U_C = X_C \cdot I_0 \cdot \cos\left(\omega \cdot t + \phi_s - \frac{\pi}{2}\right) \quad (VI) \]
at the capacitor. The sum of these two voltages is
\[ U_S = \sqrt{R^2 + X_C^2} \cdot I_0 \cdot \cos(\omega \cdot t) \quad (VII) \]
if \( \phi_s \) fulfills the condition
\[ \tan \phi_s = \frac{X_C}{R} \quad (VIII). \]

\( U_S \) is equal to the voltage \( U \) applied, and therefore
\[ U_0 = \sqrt{R^2 + X_C^2} \cdot I_0 \quad (IX), \]
i.e. the series connection of an ohmic resistor and a capacitor can be assigned the impedance

![Fig. 1 AC circuit with a capacitor (circuit diagram, vector diagram and \( U(t), I(t) \) diagram)]
**Apparatus**

1 plug-in board A4  
1 resistor 1 Ω, 2 W, STE 2/19  
1 resistor 100 Ω, 2 W, STE 2/19  
1 capacitor 0.1 µF, 100 V, STE 2/19  
1 capacitor 1 µF, 100 V, STE 2/19  
1 capacitor 10 µF, 100 V, STE 2/19  
1 function generator S 12  
1 two-channel oscilloscope 303  
2 screened cables BNC/4 mm  
Connecting leads

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The sum of the two currents is

\[ I_p = \frac{1}{\sqrt{R^2 + X_C^2}} \cdot U_0 \cdot \cos(\omega t + \varphi_p) \quad \text{(XIII)} \]

with

\[ \tan \varphi_p = \frac{R}{X_C} \quad \text{(XIV)} \]

It corresponds to the total current drawn from the voltage source. Hence, the parallel connection of an ohmic resistor and a capacitor can be assigned an impedance \( Z_p \), for which

\[ \frac{1}{Z_p} = \frac{1}{\sqrt{R^2 + X_C^2}} \quad \text{(XV)} \]

holds. In this arrangement, the current is phase-shifted by the angle \( \varphi_p \) with respect to the voltage (see Fig. 3).

In the experiment, the current \( i(t) \) and the voltage \( u(t) \) are measured as time-dependent quantities in an AC circuit by means of a two-channel oscilloscope. A function generator is used as a voltage source with variable amplitude \( U_0 \) and variable frequency \( f \). From the measured quantities the magnitude of the total impedance \( Z \) and the phase shift \( \varphi \) between the current and the voltage are determined.

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**Parallel connection**

If the capacitor is connected in parallel to the ohmic resistor, the same voltage is applied to both of them. The voltage has, for example, the shape given in Eq. (I). The current flowing through the ohmic resistor is

\[ I_R = \frac{U_0}{R} \cdot \cos(\omega t) \quad \text{(XI)} \]

whereas the current flowing through the capacitor is

\[ I_C = \frac{U_0}{X_C} \cdot \cos(\omega t + \frac{\pi}{2}) \quad \text{(XII)} \]

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**Fig. 2** AC circuit with a capacitor and an ohmic resistor in series connection (circuit diagram, vector diagram and \( U(t), i(t) \) diagram)

**Fig. 3** AC circuit with a capacitor and an ohmic resistor in parallel connection (circuit diagram, vector diagram and \( U(t), i(t) \) diagram)
Setup
The experimental setup is illustrated in Fig. 4.

- Connect the function generator as an AC voltage source, and select the shape \( \wedge \).
- Connect the channel I of the oscilloscope to the output of the function generator, and feed the voltage drop at the measuring resistor into the channel II.
- Press the DUAL pushbutton at the oscilloscope, and select AC for the coupling and the trigger.

![Experimental setup for determining the impedance in circuits with capacitors and ohmic resistors in series connection (above) and in parallel connection (below)](image)

Carrying out the experiment

- Connect the 10 \( \mu \text{F} \) capacitor as a capacitance in series with the 100 \( \Omega \) resistor.
- Switch the function generator on by plugging in the plug-in power supply, and adjust a frequency of 2000 Hz (\( T = 0.5 \) ms).
- Select an appropriate time-base sweep at the oscilloscope.
- Adjust an output signal of 5 V.
- Read the amplitude of the signal in the channel II of the oscilloscope, and enter it in the table as current \( I_0 = \frac{U_m}{1 \Omega} \).
- Read the time difference \( \Delta t \) between the zero passages of the two signals.
- Connect the 10 \( \mu \text{F} \) capacitor in parallel to the 100 \( \Omega \) resistor.
- Repeat the measurement.
- Repeat the measurements with the 1 \( \mu \text{F} \) capacitor and with the 0.1 \( \mu \text{F} \) capacitor.
- Adjust other frequencies according to Table 1, and repeat the measurements.

Measuring example

\( U_0 = 5.0 \text{ V}, R_m = 1 \Omega, R = 100 \Omega \)

Table 1: measuring data for the frequency \( f \), oscillation period \( T \), capacitance \( C \), time difference \( \Delta t \) and current amplitude \( I_0 \)

<table>
<thead>
<tr>
<th>( f ) (Hz)</th>
<th>( T ) (ms)</th>
<th>( C ) (( \mu \text{F} ))</th>
<th>( I_0 ) (mA)</th>
<th>( \Delta t ) (ms)</th>
<th>( I_0 ) (mA)</th>
<th>( \Delta t ) (ms)</th>
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<td>48</td>
<td>0.01</td>
<td>620</td>
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<td>0.06</td>
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<td></td>
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<td>330</td>
<td>0.21</td>
</tr>
<tr>
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<td>27</td>
<td>0.16</td>
<td>58</td>
<td>0.09</td>
</tr>
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<td></td>
<td></td>
<td>0.1</td>
<td>3.5</td>
<td>0.23</td>
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<td>0.01</td>
</tr>
<tr>
<td>500</td>
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<td>10</td>
<td>46</td>
<td>0.10</td>
<td>170</td>
<td>0.38</td>
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<tr>
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<td>10</td>
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<td>60</td>
<td>0.90</td>
</tr>
<tr>
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<td></td>
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<td>3.5</td>
<td>2.4</td>
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<td>0.10</td>
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<tr>
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<td>10</td>
<td>15</td>
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</table>
Evaluation

The measuring data of Table 1 are evaluated as follows:

The phase shift $\phi$ is calculated from the time difference $\Delta t$ between the voltage and the current and from the oscillation period $T$ according to

$$\phi = 360 \frac{\Delta t}{T}$$

and the magnitude of the total impedance is obtained from the amplitudes $U_0$ and $I_0$ according to

$$Z = \frac{U_0}{I_0}.$$  

The results are listed in Table 2, where the capacitive reactance of the respective capacitor calculated according to Eq. (III) is also given.

For the series connection, Fig. 5 shows a plot of the impedance $Z_S$ and Fig. 6 shows the phase shift $\phi_S$ between the current and the voltage, both as functions of the capacitive reactance $X_C$. The solid lines were calculated according to Eq. (X) and Eq. (VIII), respectively.

The corresponding diagrams for the parallel connection are shown in Figs. 7 and 8. In this case, the solid curves are obtained according to Eqs. (XIV) and (XV).

Table 2: values of the total impedance $Z$ and the phase shift $\phi$ between the current and the voltage calculated from the measuring data from Table 1

<table>
<thead>
<tr>
<th>$f$ (Hz)</th>
<th>$\frac{C}{\mu F}$</th>
<th>$\frac{X_C}{\Omega}$</th>
<th>$\frac{Z}{\Omega}$</th>
<th>$\phi$</th>
<th>$\frac{Z}{\Omega}$</th>
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<td>79°</td>
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<td>61</td>
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</tr>
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<td></td>
<td>0.1</td>
<td>800</td>
<td>830</td>
<td>86°</td>
<td>100</td>
<td>7°</td>
</tr>
<tr>
<td>1000</td>
<td>10</td>
<td>16.0</td>
<td>104</td>
<td>11°</td>
<td>15.2</td>
<td>76°</td>
</tr>
<tr>
<td></td>
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<td>160</td>
<td>185</td>
<td>58°</td>
<td>86</td>
<td>32°</td>
</tr>
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<td></td>
<td>0.1</td>
<td>1600</td>
<td>1430</td>
<td>83°</td>
<td>104</td>
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</tr>
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<td>330</td>
<td>68°</td>
<td>95</td>
<td>18°</td>
</tr>
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</table>

Fig. 5 Total impedance $Z_S$ of the series connection of a capacitor and a 100 $\Omega$ resistor as a function of the capacitive reactance $X_C$.

Fig. 6 Phase shift $\phi_S$ between the current and the voltage for the series connection of a capacitor and a 100 $\Omega$ resistor as a function of the capacitive reactance $X_C$. 
Fig. 7 Total impedance $Z_S$ of the parallel connection of a capacitor and a 100 $\Omega$ resistor as a function of the capacitive reactance $X_C$.

Fig. 8 Phase shift $\phi$ between the current and the voltage for the parallel connection of a capacitor and a 100 $\Omega$ resistor as a function of the capacitive reactance $X_C$.

Supplementary information

The mathematical description of the series and parallel connection of an ohmic resistance and a capacitive reactance becomes more elegant if complex quantities are considered.

When a voltage

$$U = U_0 \cdot e^{j\omega t}$$

is applied to a capacitor, the capacitive reactance is

$$X_C = \frac{1}{j \cdot \omega \cdot C}.$$  

The impedance $Z_S$ of a series connection of an ohmic resistance $R$ and a capacitive reactance then is

$$Z_S = R + \frac{1}{j \cdot \omega \cdot C}.$$  

In the case of a parallel connection, the following relation holds for the total impedance $Z_P$:

$$\frac{1}{Z_P} = \frac{1}{R} + j \cdot \omega \cdot C.$$