Determining the capacitance of a plate capacitor

Measuring the charge with the I-Measuring Amplifier D

**Objects of the experiment**
- Measuring the charge as function of the applied voltage
- Measuring the charge as function of the plate spacing
- Determination of the capacitance

**Principles**

If an electric conductor carries the charge +Q and on another electric conductor is charged by the charge −Q, then a potential difference \( U \) exists between these conductors:

\[
Q = C \cdot U \quad (\text{I})
\]

The proportionality factor \( C \) is called capacitance. The capacitance \( C \) depends on the geometrical arrangement of the conductors and the non-conducting material between these conductors. The arrangement of the conductors is called capacitor. The simplest design of a capacitor is that of two parallel plates. The capacitance of a plate capacitor is given by

\[
C = \varepsilon_r \cdot \varepsilon_0 \frac{A}{d} \quad (\text{II})
\]

- \( \varepsilon_r = 8.85 \cdot 10^{-12} \ \text{A}^2 \text{V}^{-1} \text{m} \) permittivity of free space,
- \( \varepsilon_r \): permittivity due to the material
- \( A \): area of the plates
- \( d \): distance between the plates

Equation (II) holds only as long as the distance between the plates is much smaller than the dimensions of the plates and the electric field \( E \) between the plates can be considered to be homogeneous. The permittivity \( \varepsilon_r \) describes the change of the capacitance relatively to the vacuum value caused by the introduction of the material.

In this experiment relation (I) is studied. The charge is measured by the I-Measuring Amplifier D as function of the applied voltage \( U \) for various plate distances \( d \). The capacitance \( C \) is then determined as the slope of the straight line through the origin and through the data points. Additionally, the variation of the distance \( d \) between the plates allows to confirm the proportionality

\[
C = \frac{1}{d}
\]
Carrying out the experiment

a) Measuring the charge as function of voltage
- Set the plates of the capacitor $d = 2$ mm apart.
- To charge the plates apply a voltage of $50$ V to the setup as depicted in Fig. 1.
- Ground the I-Measuring amplifier D with the zero button ⚪ (see Fig. 1 and also instruction sheet 532 00).
- Disconnect the red cable from the fixed plate and connect the coaxial cable BNC/4mm to the plate (Fig. 2).

Note: Instead of disconnecting the red cable from the fixed plate and connecting the coaxial cable BNC/4mm in its place an alternative setup may be used under certain circumstances. This setup is described in chapter supplementary information.
- Measure the output voltage of the I-Measuring amplifier D and determine the charge. Repeat this step several times and determine the average values.
- Repeat the measurement for other voltages, e.g. $100$ V, $150$ V, $200$ V, $250$ V and $300$ V.
- Repeat the measurement series for different plate distances $d$.

b) Measuring the charge as function of plate spacing
- Set the plates of the capacitor $d = 2$ mm apart.
- To charge the plates apply a voltage, e.g. $100$ V, to the setup depicted in Fig. 1.
- Ground the I-Measuring amplifier D with the zero button ⚪ (see Fig. 1 and also instruction sheet 532 00).
- Disconnect the red cable from the fixed plate and connect the coaxial cable BNC/4mm to the plate (Fig. 2).
- Measure the output voltage of the I-Measuring amplifier D and determine the charge. Repeat this step several times and determine the average values.
- Repeat the measurement for the plate distances $4$ mm, $6$ mm, $8$ mm, $10$ mm and $12$ mm.

Setup

Set up the experiment as shown in Fig. 1.

Note: The movable plate is connected to the earth socket of the power supply and the I-Measuring amplifier D. The isolated plate is connected via the resistor $100 \text{ M}\Omega$ to the positive socket of the power supply.

For measuring the charge on the capacitor the I-Measuring Amplifier D is switched to the range $10^{-8}$ As. The $3$ V or $10$ V DC range may be selected at multimeter. Then, for example, an output voltage of $3$ V corresponds to $3\cdot10^{-8}$ As.
Measuring example

a) Measuring the charge as function of voltage

Table 1: Charge Q (average over 3 measurements) as function of the applied voltage U for fixed plate distances d.

<table>
<thead>
<tr>
<th>U / V</th>
<th>Q / nAs (d = 2 mm)</th>
<th>Q / nAs (d = 4 mm)</th>
<th>Q / nAs (d = 6 mm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>50</td>
<td>12.0</td>
<td>6.8</td>
<td>4.5</td>
</tr>
<tr>
<td>100</td>
<td>24.0</td>
<td>13.0</td>
<td>9.0</td>
</tr>
<tr>
<td>150</td>
<td>35.5</td>
<td>19.7</td>
<td>13.9</td>
</tr>
<tr>
<td>200</td>
<td>48.0</td>
<td>26.5</td>
<td>18.3</td>
</tr>
<tr>
<td>250</td>
<td>61.5</td>
<td>33.3</td>
<td>23.0</td>
</tr>
<tr>
<td>300</td>
<td>74.0</td>
<td>40.0</td>
<td>27.9</td>
</tr>
</tbody>
</table>

b) Measuring the charge as function of plate spacing

Table 2: Charge Q (average over 3 measurements) as function of the plate distances d for the applied voltage U = 100 V. The capacitance is determined by equation (I).

<table>
<thead>
<tr>
<th>d / mm</th>
<th>Q / nAs</th>
<th>C / pF</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>24.0</td>
<td>240</td>
</tr>
<tr>
<td>4</td>
<td>12.8</td>
<td>128</td>
</tr>
<tr>
<td>6</td>
<td>9.0</td>
<td>90</td>
</tr>
<tr>
<td>8</td>
<td>7.1</td>
<td>71</td>
</tr>
<tr>
<td>10</td>
<td>6.0</td>
<td>60</td>
</tr>
<tr>
<td>12</td>
<td>5.2</td>
<td>52</td>
</tr>
</tbody>
</table>

Evaluation and results

a) Measuring the charge as function of voltage

Fig. 3 summarizes the result of table 1. The straight lines correspond to fits according equation (I). From the slopes the capacitance is determined to:

\[ C = 244 \text{ pF} \quad (d = 2 \text{ mm}) \]

\[ C = 133 \text{ pF} \quad (d = 4 \text{ mm}) \]

\[ C = 92 \text{ pF} \quad (d = 6 \text{ mm}) \]

b) Measuring the charge as function of plate spacing

Fig. 4 summarizes the result of table 2. In accordance with equation (II) the capacitance decreases with increasing plate distance d.

In Fig. 5 the capacitance is plotted versus the inverse values of the plate distance d to confirm the proportionality:

\[ C \propto \frac{1}{d} \]

The deviations are due additional capacities in measuring setup and an increasing region of the inhomogeneous electric field on the bothers of the plates.
Performing a fit according equation (II) the permittivity $\varepsilon_0$ ($\varepsilon_r \approx 1$) can be determined from the slope (Fig. 5). Taking into account the inevitable additional capacities (order of magnitude 15 pF) the slope has been determined to:

Slope: $s = 451 \cdot \text{pF} \cdot \text{mm}$

Area of the plates: $A = 5.1 \cdot 10^{-2} \text{m}^2$

$\varepsilon_0 = \frac{8.83 \cdot 10^{-12} \text{As}}{\text{Vm}}$

Supplementary information

Using the two-way switch (504 48) the two steps charging the capacitor (Fig. 1) and measuring the charge on the capacitor (Fig. 2) can done in one step conveniently. However, this setup depicted in Fig 6 may not work properly at ambient conditions with high humidity due to leakage currents.

Fig. 6: Alternative experimental setup (wiring diagram schematically), compare Fig. 1 and Fig. 2, respectively.