Measuring the force between a charged sphere and a metal plate

**Principles**

By electrostatic induction (displacement of charges), a pointlike charge \( Q \) at a distance \( d \) from an earthed metal plate generates an excess charge of the opposite sign on the surface of the metal plate. The charge \( Q \) is, therefore, acted upon by an attractive force towards the metal plate.

This attractive force \( F \) is equal to the force that a pointlike charge \(-Q\) at a distance \( 2d \) would exert on the charge \( Q \), that is,

\[
F = \frac{1}{4\pi \varepsilon_0} \cdot \frac{-Q^2}{(2d)^2}
\]

This relation is illustrated in Fig. 1: In the case of an equilibrium, the electric field lines starting from \( Q \) meet the metal plate perpendicularly because a field component parallel to the surface of the plate would lead to a displacement of the charge distribution on the metal plate, which cannot occur in a state of equilibrium. The same shape of the field lines is generated by a “mirror” charge \(-Q\) placed at the mirror image of the point where \( Q \) is located with respect to the plate.

In the experiment, the force between a charged sphere and a metal plate is measured with the aim of confirming the proportionality

\[
F \propto \frac{1}{d^2}
\]

and

\[
F \propto Q^2
\]

In the experiment, however, the mutual electrostatic induction between the sphere and the metal plate leads to a displacement of charges on the sphere as well – which shows up particularly at small distances \( d \). This displacement has the same effect that a reduction of the distance \( d \) would have, and it leads to an enhancement of the force \( F \).
Preliminary remarks
Carrying out this experiment requires particular care because “leakage currents” through the insulators may cause charge losses and thus considerable measuring errors. Moreover, undesirable effects of electrostatic induction may influence the results.

The experiment must be carried out in a closed, dry room so as to prevent charge losses due to high humidity.

Cleaning the insulators of the spheres with distilled water is recommended because distilled water is the best solvent of conductive salts on the insulators. In addition, the insulators should be discharged before every experiment by passing them several times quickly through a non-blackening flame, for example of a butane gas burner. The plastic rod and the leather too must be dry and clean.

Carrying out the experiment
Notes:
The measurement is susceptible to impact through interferences from the vicinity because the forces to be measured are very small: Avoid vibrations, draught and variations in temperature.

The newtonmeter must warm up at least 30 min before the experiment is started: the force sensor being connected, switch the newtonmeter on at the mains switch on the back of the instrument.

a) The force $F$ as a function of the distance $d$:  
- Make the zero compensation by setting the pushbutton COMPENSATION of the newtonmeter to SET. 
- Charge the plastic rod by rubbing it with the leather. 
- Charge the sphere (a) by touching it with the plastic rod. 
- Read the force $F$ and record it together with the distance $d$. 
- Increase the distance $d$ with the height adjustment screw (c) in steps of 2.5-mm up to $d = 45$ mm. In each case, read the force from the newtonmeter and record it together with the distance.

b) The force $F$ as a function of the charge $Q$:  
- Set the distance $d$ to 35 mm. 
- Make the zero compensation of the newtonmeter again. 
- Recharge the plastic rod by rubbing it with the leather, and charge the sphere (a) by touching it with the plastic rod. 
- Read the force from the newtonmeter and record it as $F(Q)$. 
- Halve the charge by touching the sphere (a) with the sphere (e), which has the same size. 
- Read the force again and record it as $F(Q/2)$. 
- Calculate the ratio of the two forces. 
- Discharge the two sphere at the capacitor plate, and repeat the experiment several times.
Measuring example

a) Measuring the force $F$ as a function of the charge $Q$:

Table 1: The force $F$ as a function of the distance $d$

<table>
<thead>
<tr>
<th>$d$ [mm]</th>
<th>$F$ [mN]</th>
</tr>
</thead>
<tbody>
<tr>
<td>30</td>
<td>−5.9</td>
</tr>
<tr>
<td>32.5</td>
<td>−4.3</td>
</tr>
<tr>
<td>35</td>
<td>−3.1</td>
</tr>
<tr>
<td>37.5</td>
<td>−2.4</td>
</tr>
<tr>
<td>40</td>
<td>−1.9</td>
</tr>
<tr>
<td>42.5</td>
<td>−1.6</td>
</tr>
<tr>
<td>45</td>
<td>−1.3</td>
</tr>
</tbody>
</table>

b) Measuring the force $F$ as a function of the charge $Q$:

Table 2: The force $F$ measured with the total charge $Q$ and with the halved charge ($d = 35$ mm)

<table>
<thead>
<tr>
<th>$n$</th>
<th>$F(Q)$ [mN]</th>
<th>$F(Q/2)$ [mN]</th>
<th>$F(Q/2)/F(Q)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>−3.2</td>
<td>−0.9</td>
<td>0.28</td>
</tr>
<tr>
<td>2</td>
<td>−5.0</td>
<td>−1.2</td>
<td>0.24</td>
</tr>
<tr>
<td>3</td>
<td>−3.4</td>
<td>−0.8</td>
<td>0.24</td>
</tr>
<tr>
<td>4</td>
<td>−4.0</td>
<td>−1.2</td>
<td>0.30</td>
</tr>
<tr>
<td>5</td>
<td>−3.0</td>
<td>−0.7</td>
<td>0.23</td>
</tr>
<tr>
<td>6</td>
<td>−3.6</td>
<td>−0.9</td>
<td>0.25</td>
</tr>
<tr>
<td>7</td>
<td>−4.6</td>
<td>−1.1</td>
<td>0.24</td>
</tr>
<tr>
<td>8</td>
<td>−3.7</td>
<td>−1.0</td>
<td>0.27</td>
</tr>
<tr>
<td>9</td>
<td>−3.9</td>
<td>−0.9</td>
<td>0.23</td>
</tr>
<tr>
<td>10</td>
<td>−4.2</td>
<td>−1.0</td>
<td>0.24</td>
</tr>
</tbody>
</table>

Evaluation

a) The force $F$ as a function of the distance $d$:

In Fig. 3, the measuring values of Table 1 are plotted. The attractive force $F$ between the charged sphere and the earthed metal plate (turns out to non-linearly) when the distance $d$ is increased.

Fig. 4 shows the measuring values in the linearized form $|F|^{1/2} = f(d)$. The measuring values lie to good approximation on a straight line, which, however, does not go through the origin, but intersects the x-axis at $d_0 = 17$ mm. Thus the proportionality $|F|^{1/2} \propto d - d_0$ or $F \propto \frac{1}{(d - d_0)^2}$ is found. This effect corresponds to a shift of the distance, which goes back to a charge displacement on the sphere towards the metal plate due to electrostatic induction.

b) Measuring the force $F$ as a function of the charge $Q$:

According to Eq. (III), the force should be reduced by a factor of four after the charge $Q$ has been halved, that is, one expects $F(Q/2)/F(Q) = 0.25$ (IV).

The mean value of the result in Table 2 is $\bar{x} = 0.252$, and the standard deviation is $\sigma = 0.008$. Thus (IV) is confirmed.

Results

An attractive force arises between a charged sphere and an earthed metal plate due to electrostatic induction. This force is equal to the force exerted by a mirror charge.