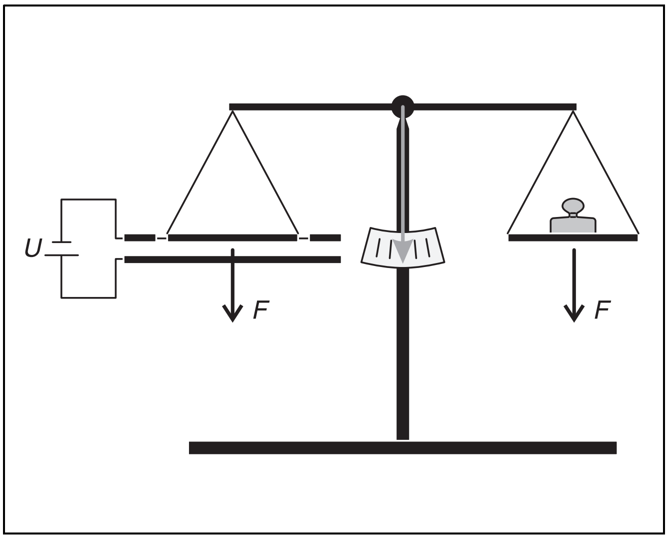
Electricity

Electrostatics
Effects of force in an electric field

Kirchhoff's voltage balance: Measuring the force between two charged plates of a plate capacitor

Objects of the experiments

- Measuring the force F between the charged plates as a function of the voltage U at a constant distance d between the plates.
- Determining the permittivity of free space ε_0.
- Measuring the force F between the charged plates at a constant ratio between the voltage U and the distance d.



Principles

A voltage U applied to a plate capacitor gives rise to a homogeneous electric field

$$E = \frac{U}{d} \qquad \text{(I)},$$

d: distance between the plates,

between the plates. This field is generated by the charges Q and $-Q$ on the capacitor plates. On the other hand, the field exerts a force on the charges. However, the more the field penetrates into the plate, the more it is attenuated. On the surface of the plates the field strength is E, but inside it is zero. On average, only half the field strength $E/2$ acts on the charges. Therefore, the plates attract each other with the force

$$F = -\frac{1}{2} \cdot Q \cdot E \qquad \text{(II)}.$$

The charge Q on the plates is

$$Q = \varepsilon_0 \cdot \frac{A}{d} \cdot U \qquad \text{(III)},$$

$\varepsilon_0 = 8.85 \cdot 10^{-12} \frac{\text{As}}{\text{Vm}}$: permittivity of free space,
A: area of the plates.

Thus from (I) and (II)

$$F = -\frac{1}{2} \cdot \varepsilon_0 \cdot A \cdot \left(\frac{U}{d}\right)^2 \qquad \text{(IV)}$$

follows. F, A, d and U are directly measurable quantities. Eq. (IV) can, therefore, be considered as the determining equation of the permittivity of free space ε_0. This is the principle of Kirchhoff's voltage balance, the setup of which is the object of the present experiment. The proportionality

$$F \propto \frac{U^2}{d^2} \qquad \text{(V)},$$

which is stated by Eq. (IV), will be confirmed experimentally.

1009-Wei

Apparatus

1 electrostatic accessories for current balance	516 37
1 vertically adjustable stand	516 31
1 newtonmeter	314 251
1 force sensor	314 261
1 support for conductor loops	314 265
1 multicore cable 6-pole, 1.5 m long	501 16
1 high voltage power supply 10 kV	521 70
1 high voltage cable, 1 m	501 05
1 stand rod, 47 cm	300 42
1 stand base, V-shape, 20 cm	300 02
1 Leybold multiclamp	301 01
connection leads	

Safety notes

The high voltage power supply 10 kV fulfills the safety requirements for electrical equipment for measurement, control and the laboratory. It supplies a non-hazardous contact voltage. Observe the following safety measures.

- Observe the instructions of the high voltage power supply.
- Always make certain that the high voltage power supply is switched off before altering the connections in the experimental setup.
- Set up the experiment so that neither non-insulated parts nor cables and plug can be touched inadvertently.
- Always set the output voltage to zero before switching on the high voltage power supply (turn the knob all the way to the left).
- In order to avoid high-voltage arcing, lay the high voltage cable in a way that there are no conductive objects near the cable.

Setup

The experimental setup is illustrated in Fig. 1. The plate capacitor consists of the capacitor plate on an insulator, the capacitor plate with a pair of plugs, and the screening ring from the set of electrostatic accessories (516 37).

Mechanical setup:

- Set the screening ring **(a)** with the stand up.
- Set the stand rod up in the stand base, and attach the force sensor (+F direction upward) to the stand rod with the Leybold multiclamp.
- Connect the force sensor to the newtonmeter with the multicore cable.
- Attach the support for conductor loops to the force sensor, connect the capacitor plate with the pair of plugs **(b)**, and align it concentrically with the screening ring without contact.
- Put the capacitor plate on the insulator **(c)** onto the vertically adjustable stand, lock with the knurled screw **(d)**, and align the plate **(c)** parallel to the capacitor plate **(b)** by means of the leveling screws **(f)**.
- Check the adjustment, and set the distance d to 20 mm by means of the adjusting screw **(e)**.

Electrical setup:

- Connect the capacitor plate **(c)** to the positive pole of the high voltage power supply, plugging the high voltage cable into the 4-mm hole in the socket of the plate.
- Connect the screening ring **(a)** to the capacitor plate **(b)**, then connect both to the negative pole of the high voltage power supply, plugging the connection lead into the 4-mm hole in the stand or into the support for conductor loops.
- Connect the negative pole to the earth of the of the high voltage power supply.
- Connect the high voltage power supply 10 kV to the mains and switch it on.

Carrying out the experiment

Notes:

The measurement is susceptible to impact through interferences from the vicinity because the forces to be measured are very small: Avoid vibrations, draught and variations in temperature.

The newtonmeter must warm up at least 30 min before the experiment is started: the force sensor being connected, switch the newtonmeter on at the mains switch on the back of the instrument.

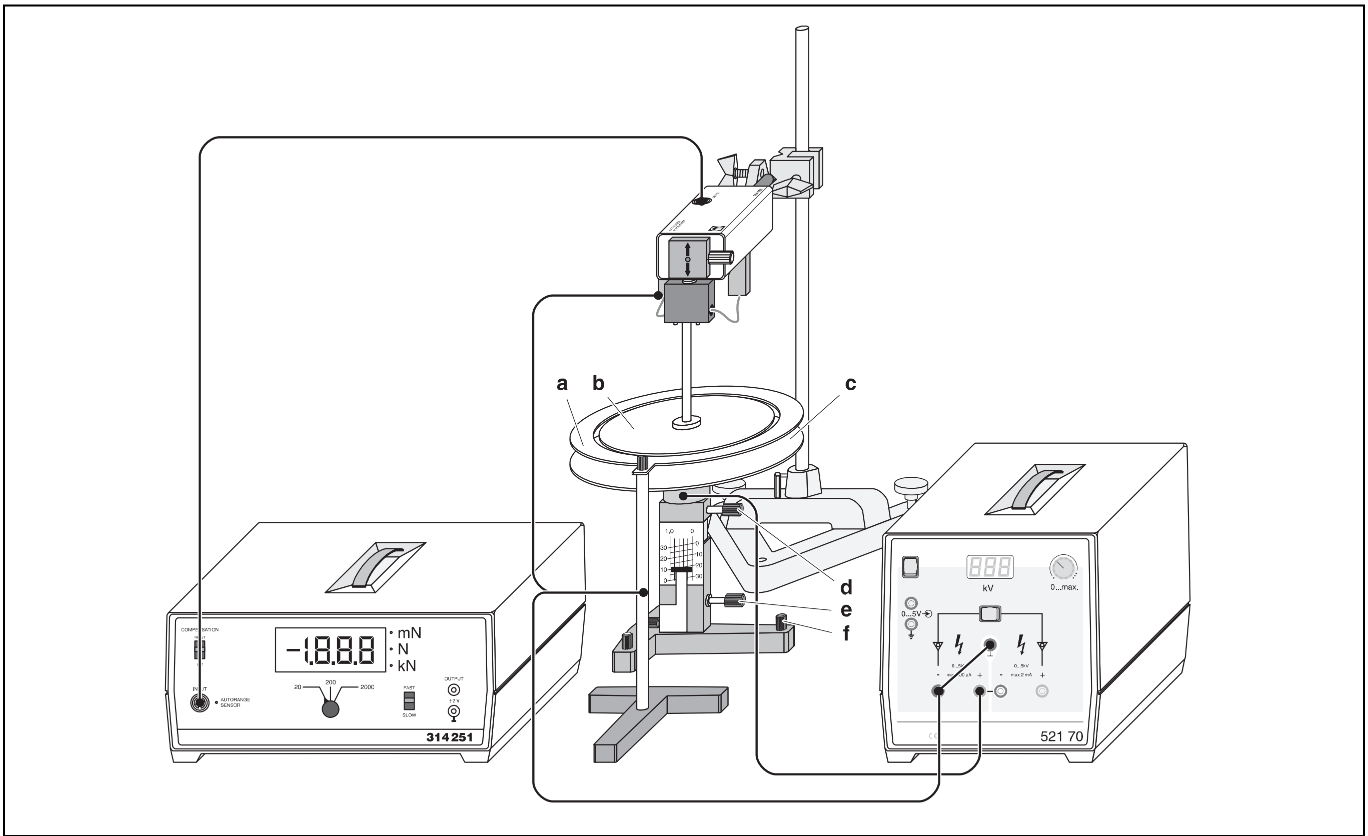


Fig. 1 Experimental setup for Kirchhoff's voltage balance

a) The force *F* as a function of the voltage *U*:

– Make zero compensation by setting the pushbutton COMPENSATION of the newtonmeter to SET.
– Switch on the high voltage power supply, and set the output voltage to U = 2 kV.
– Read the force F from the newtonmeter and record it.
– Increase the high voltage in steps of 0.5 kV up to 5 kV. In each case, read the force F and record it together with the voltage U.

b) The force *F* at a constant ratio between the voltage *U* and the distance d between the plates:

– Set the high voltage back to zero, and make zero compensation of the newtonmeter again.
– Set the high voltage to U = 5 kV and read the force F from the newtonmeter.
– Set the high voltage to U = 4 kV, and reduce the distance d between the plates to 16 mm; make sure that the capacitor plates and the screening ring do not touch.
– Read the force F from the newtonmeter and record it together with the values U, d and .
– Repeat the measurement at U = 3 kV, d = 12 mm and at U = 2 kV, d = 8 mm.

Measuring example

a) The force *F* between the charged plates as a function of the voltage *U*:

radius of the capacitor plate: 15 cm

Table 1: The force F between the charged plates as a function of the high voltage U (d = 20 mm)

$\frac{U}{\text{kV}}$	$\frac{F}{\text{mN}}$
5.0	–4.0
4.5	–3.2
4.0	–2.5
3.5	–2.0
3.0	–1.5
2.5	–1.0
2.0	–0.6

b) The force *F* at a constant ratio between the voltage *U* and the distance d between the plates:

Table 2: The force F between the charged plates at a constant ratio $E = U/d$

$\frac{U}{\text{kV}}$	$\frac{d}{\text{mm}}$	$\frac{E}{10^6 \frac{\text{V}}{\text{m}}}$	$\frac{F}{\text{mN}}$
5.0	20	0.25	–4.0
4.0	16	0.25	–4.0
3.0	12	0.25	–4.0
2.0	8	0.25	–4.0

Evaluation

a) The force *F* as a function of the voltage *U*:

Fig. 2 is a plot of the measuring values from Table 1. The curve drawn in is a parabola with its maximum in the origin. It can be seen that the attractive force increases with the square of the high voltage *U,* that is,

$F \propto U^2$.

In Fig. 3, the measuring values are plotted in the linearized form $F = f(U^2)$. The data points lie to a good approximation on a straight line through the origin.

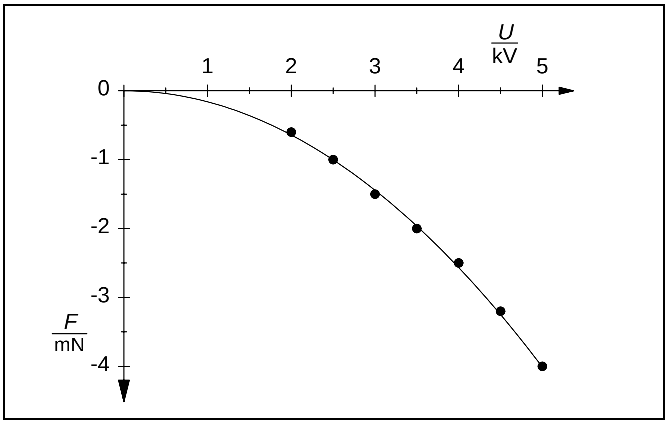


Fig. 2 The force *F* between the charged plates as a function of the voltage *U* at a constant distance *d* = 20 mm between the plates.

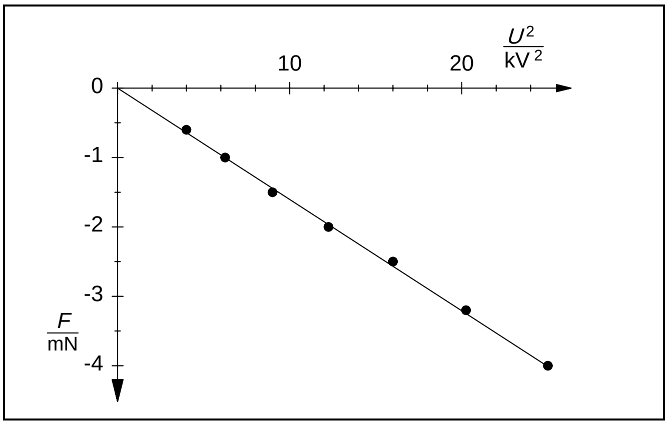


Fig. 3 Graph of the measuring values of Fig. 2 in the linearized form $F = f(U^2)$

Determining the permittivity of free space ε_0:

According to Eq. (IV), the permittivity of free space can be determined from the slope

$$\frac{F}{U^2} = -0.16\,\frac{\text{mN}}{\text{kV}^2}$$

of the straight line through the origin in Fig. 4:

$$\varepsilon_0 = -\frac{F}{U^2} \cdot \frac{2d^2}{A}.$$

The distance *d* between the plates is 20 mm.

The area *A* is calculated with the radius *r* =7.5 cm of the smaller capacitor plate since the force *F* acting on this plate has been measured: $A = 1.7 \cdot 10^{-2}\text{m}^2$.

The result is: $\varepsilon_0 = 7.5 \cdot 10^{-12}\,\frac{\text{As}}{\text{Vm}}$.

Value quoted in the literature: $\varepsilon_0 = 8.85 \cdot 10^{-12}\,\frac{\text{As}}{\text{Vm}}$.

b) The force *F* at a constant ratio between the voltage *U* and the distance d between the plates:

The measuring values of Table 2 show that the force *F* depends on the electric field strength when the voltage *U* and the distance *d* between the plates are varied. The force remains constant as long as the field strength remains constant.

From $F \propto U^2$ thus the proportionalities

$F \propto E^2$, at constant distance, and

$F \propto \frac{1}{d^2}$, at constant voltage,

follow.

Results

An attractive force, which depends quadratically on the electric field strength, acts between the charged plates of a plate capacitor.