

Heat

Kinetic theory of gases
Specific heat of gases

Physics
Leaflets

P2.5.3.1

Determining the adiabatic exponent C_p/C_V of air after Rüchardt

Objects of the experiment

- Measuring of the oscillation period of a steel ball.
- Determining the adiabatic coefficient of air.

Principles

The kinetic gas theory makes predictions about the adiabatic exponent κ of ideal gases. The internal energy U of a gas can be changed by increasing the heat quantity ΔQ from outside or performing the mechanical work on ΔW :

$$\Delta U = \Delta Q + \Delta W \quad (I)$$

For adiabatic changes of states no heat exchange takes place with the environment. As a consequence the heat quantity doesn't change: $\Delta Q = 0$ (i.e. the entropy remains constant).

With the ideal gas law $p \cdot V = m \cdot R \cdot T$ and $\Delta U = C_V \cdot m \cdot \Delta T$ follows from the equation (I) the Poisson's equation which describes the adiabatic exchange of state:

$$p \cdot V^\kappa = \text{const} \quad (II)$$

Here, the adiabatic exponent κ is given by

$$\kappa = \frac{C_p}{C_V} \quad \text{with} \quad C_p = C_V + R \quad (III)$$

and

C_p : specific heat if pressure is kept constant

C_V : specific heat if volume is kept constant

when the temperature is changed.

Theoretical predictions about C_p and C_V

In order to heat one mole of an ideal gas by ΔT keeping its volume constant (isochorical change of state) the energy

$$\Delta Q = C_V \cdot \Delta T \quad (IV)$$

is required. C_V is here the molar heat capacity. Due to the constant volume during the change of state the mechanical work doesn't change ($\Delta W = 0$) in equation (I). The supplied heat ΔQ thus increases only the kinetic energy of the particles. In accordance with the law of equipartition of the kinetic gas theory, the increase in the average kinetic energy of the particles is given by

$$\overline{\Delta E_{\text{kin}}} = \frac{1}{2} \cdot f \cdot R \cdot \Delta T \quad (V)$$

f : number of degrees of freedom per particle

R : universal gas constant

ΔT : change in temperature

From the comparison of equations (IV) and (V) follows that the molar heat capacity of an ideal gas is solely determined by the numbers of degrees of freedom of the gas molecules:

$$C_V = \frac{1}{2} \cdot f \cdot R \quad (VI)$$

The particles of a mono-atomic gas possesses exactly 3 degrees of freedom of translation ($f = 3$). The rotational degrees of freedom are not excited in mono-atomic gases by the average thermal energy (~ 25 meV) due to the quantization of the angular momentum in microphysics. Diatomic and linear poly-atomic molecules have also rotational degrees of freedom. However, due to the small axial moment of inertia diatomic and linear poly-atomic molecules rotate solely about an axis perpendicular to the molecular axis.

Linear molecules possess 2 degrees of rotational freedom in addition to the three translational degrees of freedom ($f = 5$). Only for non-linear molecules all three degrees of rotational freedom are excited at room temperature ($f = 6$).

Applying the ideal gas law allows to derive the following relation between the number of degrees of freedom and the adiabatic exponent (Table 1):

$$\kappa = \frac{C_p}{C_v} = \frac{f+2}{f} \quad \text{(VII)}$$

Table 1: Adiabatic exponents of ideal gases according equation (VII):

type of gas	f	κ
mono atomic (He, Ar)	3	1.67
linear molecules (O ₂ , N ₂ , H ₂)	5	1.40
non linear molecules	6	1.33

Determination of the ratio C_p/C_v by the Rüchardt method

In this experiment the adiabatic exponent κ of air is determined from the oscillation period of a steel ball which caps and seals a gas volume in a gas tube.

According Fig. 1. the ball inside the precision glass tube is in equilibrium if the pressure p inside the aspirator is equal to the sum of the atmospheric pressure p_0 and the pressure due to the weight of the steel ball:

$$p = p_0 + \frac{m \cdot g}{A} \quad \text{(VIII)}$$

m : mass of the steel ball

g : acceleration due to gravity

A : cross-sectional area of the glass tube.

When the ball moves a distance x beyond its equilibrium position the pressure changes by dp . By this a force $F = A \cdot dp$ is exerted on the steel ball imparting to it an acceleration by Newton's second axiom:

$$A \cdot dp = m \frac{d^2x}{dt^2} \quad \text{(IX)}$$

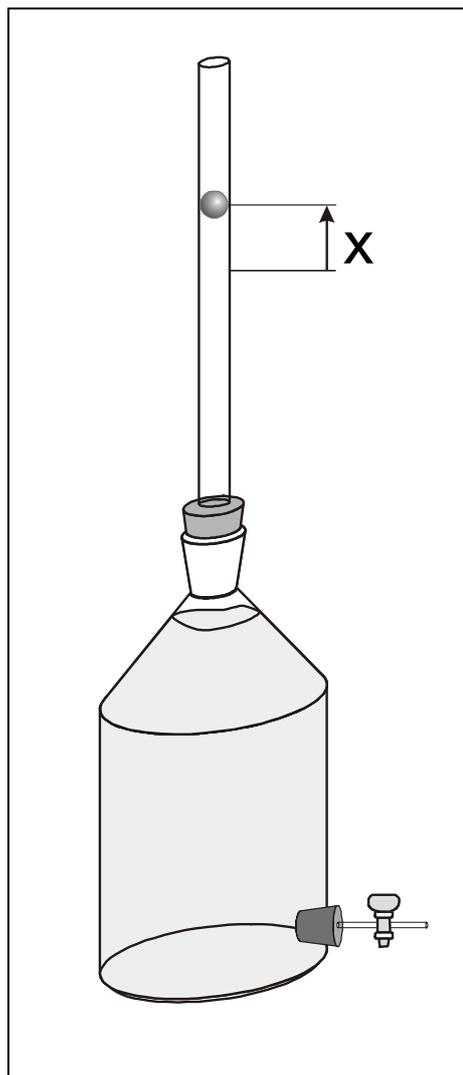


Fig. 1: Experimental setup for determining the adiabatic exponent κ after Rüchardt.

As this rapid process can be considered practically adiabatic it follows from equation (II) by differentiation:

$$V^\kappa dp + p \kappa V^{\kappa-1} dV = 0 \quad \text{(X)}$$

$$dp = -\frac{\kappa p}{V} dV \quad \text{(XI)}$$

The steel ball was supposed to move a distance x in the precision glass tube; this gives a change of volume:

$$dV = A \cdot x \quad \text{(XII)}$$

By substituting equation (XII) in equation (XI) and inserting in equation (IX) allows to rewrite the equation of motion as follows:

$$\frac{d^2x}{dt^2} + \frac{\kappa p A^2}{m V} x = 0 \quad \text{(XIII)}$$

This is the differential equation of a harmonic oscillation from which the angular frequency can be deduced:

$$\omega = \sqrt{\frac{\kappa p A^2}{m V}} \quad (\text{XIV})$$

From this relation the period of the harmonic oscillation performed by the ball is given by:

$$T = \frac{2\pi}{\omega} = 2\pi \sqrt{\frac{m V}{\kappa p A^2}} \quad (\text{XV})$$

From this equation follows for the adiabatic exponent to

$$\kappa = \frac{4\pi^2 m V}{A^2 p T^2} \quad (\text{XVI})$$

As all quantities on the right hand side of equation (XVI) are accessible to measurement the adiabatic exponent κ can be determined.

Apparatus

1 Marriot's flask, 10 l.....	371 04
1 Oscillation tube apparatus	371 05
1 Stopclock.....	313 07
1 Aneroid barometer.....	317 19
1 Plastic beaker, 1000 ml	590 06
1 Vaseline, 50 g.....	675 3100

Experimental hints

According equation (XV) the greater the enclosed volume V , the longer the oscillation period T of the steel ball. On the other hand the accuracy of the result depends on the exact determination of the oscillation period T whose square enters in equation (XVI). Therefore the aspirator is used in this experiment.

The number of oscillations the ball can perform depends crucially on the cleanness of the precision glass tube and the steel ball. Before each experiment the precision glass tube has to be carefully cleaned by drawing a soft piece of tissue or leather through it on a thread (Fig. 2). Use a detergent solution for cleaning the glass tube with the tissue.

Note

When not using the apparatus close the precision glass tube with (rubber) stoppers to keep the tube and the steel ball clean!

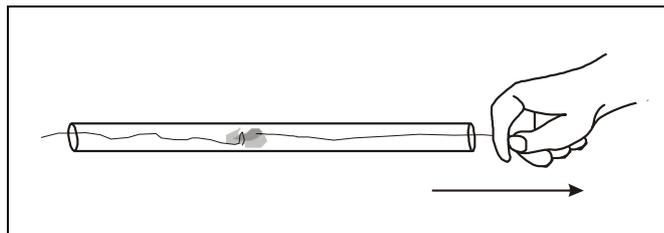


Fig. 2: Before each experiment the precision glass tube and the steel ball have to be cleaned carefully with a tissue.

Setup

- Clean the precision glass tube as depicted in Fig. 2.
- Clean the steel ball.

Note

Only a clean precision glass tube and a clean steel ball will produce sufficient oscillations.

- Insert the precision glass tube into the rubber bung. You may use a stopper ring at the other end of the rubber stopper which can be simply realized by a sheet of paper (Fig. 3).

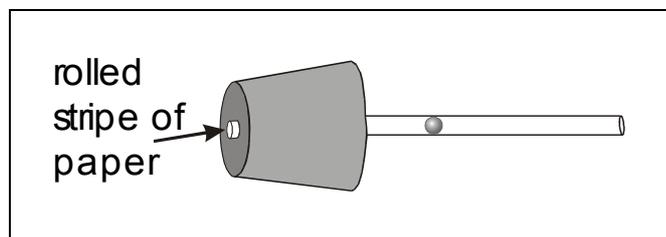


Fig. 3: For performing the experiment very conveniently a stopper ring (rolled stripe of paper) may be used in the rubber.

Carrying out the experiment

- Ensure the tap at the lower hole of the aspirator is closed.
- Insert vertically the rubber stopper with the precision glass tube into the top hole of the aspirator.
- If now the ball is allowed to fall into the glass tube it performs harmonic oscillations on the air cushion formed by the enclosed volume of air. The oscillations are damped due to the unavoidable losses of energy by friction.
- Measure the oscillation period T for e.g. 10 oscillations several times (e.g. 20 times).
- Read the pressure value from the barometer.
- Determine the volume of the aspirator by filling the vessel with water using the plastic beaker 1l. After the removing the water the vessel must be dried carefully.

Useful hints:

- For starting the experiment without touching the steel ball it might be useful to close the upper end of the precision glass tube with the hand and remove the precision glass tube with the rubber stopper from the aspirator as shown in Fig. 4.
- By letting some air into the tube let the steel ball sink down to the upper end of the precision glass tube.
- Close again the upper end of the precision glass tube and insert the precision glass tube with the rubber stopper back into the aspirator.
- Remove the hand from the precision glass tube to start the experiment.

The height of equilibrium of the steel ball in the precision glass tube (Fig. 1) depends on the pressure. If the atmospheric pressure is too low the steel ball might fall into the aspirator when releasing it at the upper end of the precision glass tube. To avoid falling into the aspirator a bar magnet might be used to chose an appropriate height for the starting position (Fig. 5).

Measuring example

Table 2: Oscillation period T of the steel ball for 10 oscillations

11.5 s	11.7 s	11.7 s	11.1 s	12.0 s
11.9 s	11.6 s	11.0 s	11.5 s	11.2 s
11.5 s	11.8 s	11.4 s	11.5 s	11.5 s
11.5 s	11.6 s	11.7 s	11.3 s	11.5 s

Volume: $V = 11,5 \text{ l}$

Evaluation and results

Evaluation of table 2 gives the mean value for the oscillation period:

$T = 1.15 \text{ s}$

With
 mass of the steel ball: $m = 16.5 \text{ g}$
 diameter of the precision glass tube: $d = 16 \text{ mm}$
 cross sectional area: $A = \pi \cdot r^2 = \pi \cdot d^2 / 4 = 2 \cdot 10^{-4} \text{ m}^2$
 pressure: $p = 1020 \text{ hPa}$
 and the volume: $V = 11.5 \text{ l}$

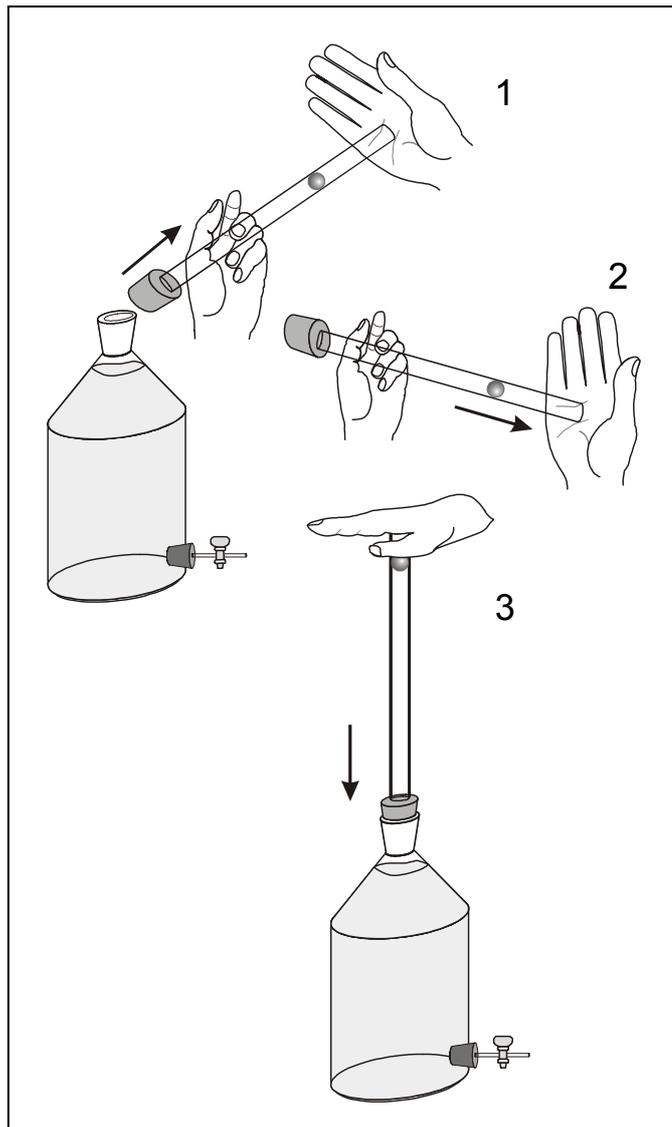


Fig. 4: Schematic representation how to restart the experiment (for details see carrying out the experiment - useful hints).

follows with equation (XVI) for the adiabatic exponent:

$\kappa = 1.37$

Literature values:

Type	O ₂	N ₂	air	CO ₂
κ	1.40	1.40	1.40	1.30

Supplementary information

The specific heat capacity and thus the adiabatic exponent depend on temperature. At low temperatures only translational degrees of freedom are excited. With increasing temperature the two degrees of rotational freedom are excited. Finally, the vibrational degrees of freedom of the molecules are excited by a further increase in temperature. Since both potential and kinetic energy occur in the oscillation, vibrational degrees of freedom must be counted double in equation (VI). As a result of the additional vibrational degree of freedom the molecular heat capacity C_V increases once more by the universal gas constant R .

The linear CO_2 molecule possesses four vibrational degrees of freedom (Fig. 5). Two normal modes occur along the molecular axis, and the other two perpendicular to the molecular axis. The bending oscillations of the CO_2 molecule are excited at room temperature and must therefore taken into account at room temperature.

This experiment can also be performed with CO_2 . The mean value of the oscillation period is determined by

$$T = 1.2 \text{ s}$$

giving an adiabatic exponent of

$$\kappa = 1.26$$

The propagation of sound waves is an adiabatic exchange of state. The local changes of pressure are so rapidly that there is no heat exchange: $\Delta Q = 0$. As a consequence the adiabatic exponent enters into the Laplace's equation for the velocity of sound in air (see experiment velocity of sound in gases P1.7.3.4).

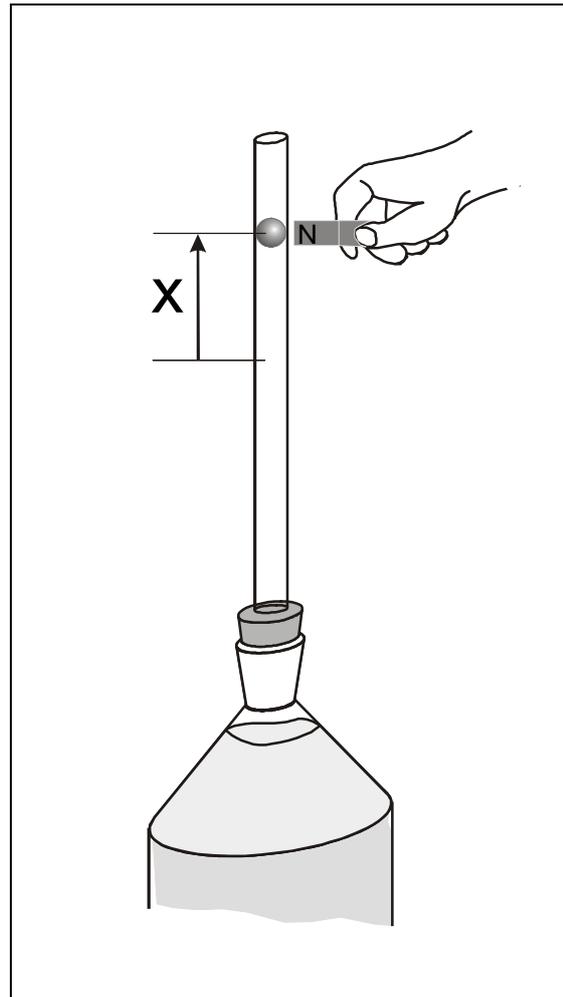


Fig. 5: Schematic representation how to use a bar magnet (not included in the equipment list) to start the experiment at an arbitrary position x (for details see carrying out the experiment - useful hints).

