Mechanics
Acoustics
Fourier analysis

Investigating fast Fourier transforms: simulation of Fourier analysis and Fourier synthesis

Description from CASSY Lab 2

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Fourier analysis of simulated signals

Experiment description

Harmonic analysis is a common method in many applications where time-variant signals (or measured values) occur. In acoustics, for example, exact knowledge of the overtones of a sound is important for the artificial generation of sounds or language.

In this experiment, the Fourier transform of simple periodic signals is studied as an introduction to the topic of Fourier transformation. In a first step, the Fourier transform of a numerically simulated signal is calculated and the frequencies of the associated amplitudes are determined (Fourier analysis). Based on this harmonic analysis, the time-variant signal is composed in a second step according to Fourier's theorem and compared with the theoretically calculated Fourier series and the numerically simulated original signal (Fourier synthesis).

Experiment setup

Remark: the experiment is a pure simulation experiment on Fourier analysis with CASSY Lab. For an experiment with electric signals of corresponding shapes see the subsequent experiment. The signals S_1 investigated in this experiment are generated by the following functions:

Delta: S_1 = 4*(1-2*saw(f*t))
Square wave: S_1 = 4*(2*square(f*t)-1)

with the frequency f = 0.5 Hz.

Remarks concerning the Fourier transformation

A continuous time-variant signal S_1 is sampled during the computer-aided measurement at certain times. In this way, a digitized signal is obtained, which can be further processed using common methods of digital signal processing (improving the signal-to-noise ratio by means of Fourier transformation, smoothing the signal by averaging, etc.). The sampling theorem tells at what time intervals the signal value has to be measured so that the time dependence of the signal can be recovered from the digitized measured values (data points). For a digitization of the signal with a sufficient number of data points, the sampling frequency f_s has to be at least twice the maximum frequency f_{max} that occurs in the signal and determines the width of the frequency spectrum. If this condition, $f_s \geq 2_{fmax}$ is not fulfilled, i.e., if the digitization takes place at a lower sampling frequency f_s, the shape of the signal is no longer captured (aliasing). The sampling frequency f_s of the measuring signal is determined by the interval $\Delta t=1/f_s$ set in the Measuring Parameters (**Window → Show Measuring Parameters**).

Fourier's theorem says that any time-dependent periodic signal S_1 can be represented by a weighted sum of sin or cos functions. For the delta and square-wave functions used in this experiment, the series expansions of S_1 in trigonometric functions up to order 9 read:

Delta:

S_3 = 4*8/3.14^2 * (cos(360*f*t) + 1/9*cos(360*3*f*t) + 1/25*cos(360*5*f*t) + 1/49*cos(360*7*f*t)) + 1/81*cos(360*9*f*t)

Square wave:

S_3 = 4*4/3.14 * (sin(360*f*t) + 1/3*sin(360*3*f*t) + 1/5*sin(360*5*f*t) + 1/7*sin(360*7*f*t) + 1/9*sin(360*9*f*t)

Thus a discrete frequency spectrum with different amplitudes corresponds to the time-dependent function S_1. The generalization of this decomposition to non-periodic signals leads to the Fourier integral, which assigns a continuous frequency spectrum F_1 to the time-dependent signal S_1.

The numerical computation of the frequency spectrum F_1 is particularly efficient if a digitized signal of $N=2^p$ data points is taken as a basis. In this case, only approx. N · log2(N) arithmetic operations have to be carried out instead of approx. N^2 operations. This procedure, which is significantly less time consuming, is called fast Fourier transformation (FFT).

CASSY Lab computes the frequency spectrum F_1 using such an algorithm. However, first of all the given measured points are weighted so that non-periodic portions on the boundaries do not play an important role (on the boundaries 0, in the middle maximum, Kaiser-Bessel window(4.0)). In order that always exactly 2^p measuring points are available, zeros are added for measuring points that may be missing.

As a result of the FFT, CASSY Lab displays a total of N/2+1 amplitudes (i.e., phase differences are not evaluated). These amplitudes are represented as "excess" amplitudes, i.e. $A_i := A_{i-1} + A_i + A_{i+1}$, in order that the amplitudes of sharp peaks are in approximate agreement with theory. Without this excess, an amplitude determination as it is carried out in this experiment would require the calculation of the sum over all amplitudes of a peak.

There are limitations to the use of FFT for frequency analysis due to two basic relations. The first one relates the highest frequency f_{max} that can be analyzed to the sampling frequency f_S:

$f_{max} = f_S/2$.

Any frequency which is greater than f_{max} appears in the frequency spectrum between zero and f_{max} and can then no longer be distinguished from the frequency contributions which really arise from the range between 0 and f_{max}. The resulting change in the signal shape is called aliasing.

The second relation relates the resolution of the frequency spectrum Δf (= distance of neighbouring points of the frequency spectrum) to the sampling frequency f_S:

$\Delta f = f_{max}/(N/2) = f_S/N = 1/\Delta t/N = 1/T$

with $T = N \cdot \Delta t$.

That means that the resolution of the frequency spectrum can only be increased by a longer measuring time.

Carrying out the experiment

- Load settings
- Using the mouse, set the pointer of the display instrument f to the desired frequency.
- simulates the recording of measured values of the function S_1. The simulation takes 50 s and records 500 values (Δt = 100 ms).

Longer recording times increase the frequency resolution of the FFT step by step whereas shorter recording times decrease the resolution.

Evaluation

The $S_1(t)$ diagram of the numerically simulated signal appears already during the simulation of the measurement. After the simulation, the Fourier transform F_1 is available in the display **Frequency Spectrum**.

The frequency spectrum exhibits peaks at odd multiples of the set signal frequency f, i.e. at f, 3*f, 5*f, 7*f, etc.. The amplitudes of the peaks can be read by clicking the curve or from the coordinate display.

Now enter the first 5 amplitudes as coefficients of the sin(360*n*f*t) functions in the Settings A1, A3, A5, A7 and A9 for the analysis. In the display **Fourier Analysis**, the time dependence of the individual terms A_1, A_3, A_5, A_7 and A_9 is shown.

In the diagram **Fourier Synthesis**, the series $S_2 = A_1 + A_3 + A_5 + A_7 + A_9$ which has been determined experimentally is compared with theoretical Fourier series S_3 and the numerically simulated function S_1. It turns out that in practical applications the periodic signal S_1 is satisfactorily approximated by a trigonometric polynomial S_2 or S_3, respectively, containing only a few terms.