Acoustic beats

Objects of the experiments
- Studying acoustic beats resulting from the superposition of tuning-fork oscillations with slightly different frequencies.
- Displaying the beats on the oscilloscope
- Determining the beat frequency $f_b$ and the frequency $f$ of the superposed oscillation and comparing these frequencies with the individual frequencies $f_1$ and $f_2$.

Principles

The wave character of sound becomes obvious when the superposition of two sound waves with equal amplitudes $A_1$ and $A_2$ and slightly different frequencies $f_1$ and $f_2$ is studied. At the position of the observer an oscillation comes about with the time dependence:

$$y(t) = A_1 \cdot \cos \left( 2\pi f_1 t + \varphi_1 \right) + A_2 \cdot \cos \left( 2\pi f_2 t + \varphi_2 \right)$$  (I).

The fact that the phases $\varphi_1$ and $\varphi_2$ of the two individual oscillations are completely arbitrary has been taken into account.

In order to calculate the beat signal, the quantities

$$A = \frac{A_1 + A_2}{2}, \quad \tilde{A} = \frac{A_1 - A_2}{2}, \quad f = \frac{f_1 + f_2}{2}, \quad \bar{f} = \frac{f_1 - f_2}{2},$$  

$$\varphi = \frac{\varphi_1 + \varphi_2}{2} \quad \text{und} \quad \varphi = \frac{\varphi_1 - \varphi_2}{2}$$

are introduced. After some transformations, the superposed signal is given by

$$y(t) = 2 \cdot A \cdot \cos \left( 2\pi \cdot \bar{f} \cdot t + \varphi \right) \cdot \cos \left( 2\pi f \cdot t + \varphi \right)$$

if the two amplitudes $A_1$ and $A_2$ agree exactly. In this case, $y(t)$ can be regarded as an oscillation with the frequency $f$ and a time dependent amplitude:

$$y(t) = a(t) \cdot \cos \left( 2\pi f \cdot t + \varphi \right)$$  (IV)

with

$$a(t) = 2 \cdot A \cdot \cos \left( 2\pi \cdot \bar{f} \cdot t + \varphi \right)$$  (V).

The magnitude of the amplitude $a(t)$ varies periodically between 0 and $2A$ (see Fig. 1), the change occurring twice during one period. The number of so-called beats per second, the beat frequency $f_b$, is therefore

$$f_b = 2 \cdot \bar{f} = f_1 - f_2$$  (VI).

When the amplitude $a(t)$ passes zero, the sign change of the beat leads to a phase jump in the superposed oscillation.

In this experiment, the superposition of two sound waves generated with tuning-forks that are slightly out of tune is studied. The beat signal is received with a microphone and then displayed on an oscilloscope. By detuning one tuning-fork the beat frequency $f_b$ is enhanced, or, in other words, the beat period

$$T_b = \frac{1}{f_b}$$  (VII)

is made shorter. The frequencies $f_1$, $f_2$, and $f_b$ are determined by measuring the corresponding periods $T_1$, $T_2$, and $T_b$ with the oscilloscope.

Fig. 1  Acoustic beats when the individual oscillations have the equal amplitudes.
**Apparatus**

<table>
<thead>
<tr>
<th>Item</th>
<th>Quantity</th>
<th>Part Number</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 pair of resonance tuning-forks, 440 Hz</td>
<td>1</td>
<td>414 72</td>
</tr>
<tr>
<td>1 multi-purpose microphone</td>
<td>1</td>
<td>586 26</td>
</tr>
<tr>
<td>1 saddle base</td>
<td>1</td>
<td>300 11</td>
</tr>
<tr>
<td>1 two-channel oscilloscope 303</td>
<td>1</td>
<td>575 21</td>
</tr>
<tr>
<td>1 BNC/4 mm adapter, 2-pole</td>
<td>1</td>
<td>575 35</td>
</tr>
</tbody>
</table>

**Setup**

The experimental setup is illustrated in Fig. 2.

- Set the switch (a) of the multi-purpose microphone to "-".
- Lower the frequency of one tuning-fork by means of the clamping screw (b).
- Put the tuning-forks on the resonance boxes, and direct the openings of the boxes towards the microphone.
- Connect the multi-purpose microphone to the oscilloscope via the BNC/4 mm adapter:
  - Zero line: middle
  - Coupling: AC
  - Scan: 20 mV/DIV.
  - Trigger: Auto
  - Time base: 20 ms/DIV.

**Carrying out the experiment**

- Strike the two tuning-forks with the hammer one immediately after the other, and compare the audible beats with the display on the oscilloscope.
- Determine the beat period $T_b$, and record it.
- Set the time base to 0.5 ms/DIV.
- In order to determine the periods $T_1$ und $T_2$, strike each tuning-fork separately while the other is removed from the resonance box.

**Measuring example**

<table>
<thead>
<tr>
<th>$T_1$ (ms)</th>
<th>$T_2$ (ms)</th>
<th>$T_b$ (ms)</th>
</tr>
</thead>
<tbody>
<tr>
<td>2.20</td>
<td>2.25</td>
<td>112</td>
</tr>
</tbody>
</table>

**Fig. 3 Experimental setup for displaying acoustic beats on the oscilloscope**

**Table 1: The individual periods $T_1$ and $T_2$ and the beat period $T_b$**

**Fig. 2 Display of an acoustic beat on the oscilloscope**
Evaluation

Table 2: The individual frequencies $f_1$ and $f_2$ and the beat frequency $f_b$

<table>
<thead>
<tr>
<th>$f_1$ Hz</th>
<th>$f_2$ Hz</th>
<th>$f_b$ Hz</th>
<th>$f_1 - f_2$ Hz</th>
</tr>
</thead>
<tbody>
<tr>
<td>455</td>
<td>444</td>
<td>9</td>
<td>11</td>
</tr>
</tbody>
</table>

Eq. (VI) is confirmed by comparing the measured beat frequency $f_b$ with the difference $f_1 - f_2$.

Results

When two acoustic oscillations with a slight difference in frequency are superposed, beats that are clearly audible are generated. These beats can be displayed on an oscilloscope.

Supplementary information

More precise investigations are possible if the beats are recorded with a storage oscilloscope or with the computer-assisted data logging system CASSY.