Inclined plane:
force along the plane and
force normal to the plane

Principles
The motion of a body on an inclined plane can be described most easily when the force exerted by the weight \( G \) (force of gravity) on the body is vectorially resolved into a force \( F_1 \) along the plane and a force \( F_2 \) normal to the plane. The force along the plane acts parallel to a plane inclined at an angle \( \alpha \), and the force normal to the plane acts perpendicular to the plane (see Fig. 1). For the absolute values of the forces, we can say

\[
F_1 = G \cdot \sin \alpha \quad \text{(I)}
\]

and

\[
F_2 = G \cdot \cos \alpha \quad \text{(II)}
\]

The experiment verifies this resolution. Here, the two forces \( F_1 \) and \( F_2 \) are measured for various angles of inclination \( \alpha \) using precision dynamometers. We can vary the angle of inclination \( \alpha \) by moving a support with the height \( h = 5 \text{ cm} \) to various distances \( s \) between the pivot of the inclined plane and the support point (see Fig. 1). We can say

\[
\sin \alpha = \frac{h}{s} \quad \text{(III)}
\]

and

\[
\cos \alpha = \sqrt{1 - \left(\frac{h}{s}\right)^2} \quad \text{(IV)}
\]

(I) and (III) give us the force along the plane

\[
F_1 = G \cdot \frac{h}{s} \quad \text{(V)}
\]

and (II) and (IV) give us the force normal to the plane

\[
F_2 = G \cdot \sqrt{1 - \left(\frac{h}{s}\right)^2} \quad \text{(VI)}
\]

Fig. 1  Vectorial resolution of the force of gravity \( G \) into the force \( F_1 \) along the plane and the force \( F_2 \) normal to the plane on an inclined plane
Apparatus
1 Inclined plane with trolley and screw model ........................................ 341 21
1 Precision dynamometer, 1.0 N ........................................................... 314 141

Setup and carrying out the experiment

a) Correcting the zero point of the dynamometer
- Lay out dynamometer F\textsubscript{1} horizontally and correct the zero point.
- Hold dynamometer F\textsubscript{2} vertically downward and correct the zero point.

b) Determining the force of gravity
- Suspend the trolley freely from dynamometer F\textsubscript{2} using the folding metal hook and determine the weight $G$ of the trolley.

c) Determining the force along the plane and the force normal to the plane
- Set up the inclined plane and position the support (e) at $s = 50$ cm.
- Place the trolley (b) on the inclined plane and hook it to dynamometer F\textsubscript{1} (c); support the dynamometer with block (d).
- Carefully arrange dynamometer F\textsubscript{2} (a) as nearly perpendicular as possible to the inclined plane and lift the trolley until it is just barely touching the plane surface.
- Read off and write down forces $F\textsubscript{1}$ and $F\textsubscript{2}$.
- Move the ramp support (b) to the positions $s = 40, 30, 20, 15$ and $10$ cm one after another; each time arrange the dynamometer perpendicular to the inclined plane and read off and write down forces $F\textsubscript{1}$ and $F\textsubscript{2}$.

Measuring example
$h = 5$ cm
$G = 1.07$ N

Table 1: Position $s$ of ramp support and forces $F\textsubscript{1}$ and $F\textsubscript{2}$ on the inclined plane

<table>
<thead>
<tr>
<th>$s$ cm</th>
<th>$F\textsubscript{1}$ N</th>
<th>$F\textsubscript{2}$ N</th>
</tr>
</thead>
<tbody>
<tr>
<td>50</td>
<td>0.10</td>
<td>1.01</td>
</tr>
<tr>
<td>40</td>
<td>0.12</td>
<td>0.98</td>
</tr>
<tr>
<td>30</td>
<td>0.18</td>
<td>0.97</td>
</tr>
<tr>
<td>20</td>
<td>0.27</td>
<td>0.97</td>
</tr>
<tr>
<td>15</td>
<td>0.35</td>
<td>0.95</td>
</tr>
<tr>
<td>10</td>
<td>0.59</td>
<td>0.81</td>
</tr>
</tbody>
</table>

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1 Inclined plane with trolley and screw model ................................. 341 21
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Fig. 2 Experiment setup for determining the force along the plane and force to the plane
Evaluation and results

Tables 2 and 3 enable a comparison of the measured forces and those calculated using (V) and (VI). Fig. 3 shows the results plotted in a graph.

For the force normal to the plane we see a systematic deviation between the measured and calculated values. This is due to the fact that the trolley is still partially supported when the force is measured.

Table 2: Measured and calculated force $F_1$ along the plane

<table>
<thead>
<tr>
<th>$s$ (cm)</th>
<th>$\frac{F_1}{N}$ measured</th>
<th>$\frac{F_2}{N}$ calculated</th>
</tr>
</thead>
<tbody>
<tr>
<td>50</td>
<td>0.09</td>
<td>0.107</td>
</tr>
<tr>
<td>40</td>
<td>0.12</td>
<td>0.134</td>
</tr>
<tr>
<td>30</td>
<td>0.18</td>
<td>0.178</td>
</tr>
<tr>
<td>20</td>
<td>0.27</td>
<td>0.268</td>
</tr>
<tr>
<td>15</td>
<td>0.33</td>
<td>0.357</td>
</tr>
<tr>
<td>10</td>
<td>0.53</td>
<td>0.535</td>
</tr>
</tbody>
</table>

Table 3: Measured and calculated force $F_2$ normal to the plane

<table>
<thead>
<tr>
<th>$s$ (cm)</th>
<th>$\frac{F_1}{N}$ measured</th>
<th>$\frac{F_2}{N}$ calculated</th>
</tr>
</thead>
<tbody>
<tr>
<td>50</td>
<td>1.01</td>
<td>1.065</td>
</tr>
<tr>
<td>40</td>
<td>0.98</td>
<td>1.062</td>
</tr>
<tr>
<td>30</td>
<td>0.97</td>
<td>1.055</td>
</tr>
<tr>
<td>20</td>
<td>0.97</td>
<td>1.036</td>
</tr>
<tr>
<td>15</td>
<td>0.95</td>
<td>1.009</td>
</tr>
<tr>
<td>10</td>
<td>0.81</td>
<td>0.927</td>
</tr>
</tbody>
</table>

Fig. 3 Measured (solid line) and calculated (points) values for the force $F_1$ along the plane and the force $F_2$ normal to the plane.