Determination of the grating constants of the holographic grating using a He-Ne-laser

Objects of the experiment

- Determination of the grating constant in the Littrow setup.
- Determination of the grating constant for various angles.

Principles

In the experiment the lattice constant g of the holographic grating is determined by means of a He-Ne laser for which the wavelength is well known.

The interference arises in reflection:

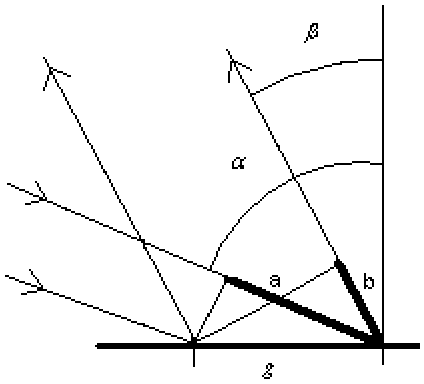


Fig. 1: For calculating the path difference

The path difference between two beams at the 1st order maximum is: $\Delta s = \mathrm{a} + \mathrm{b} = \lambda$

With an angle of incidence α and the diffraction angle β ($\frac{\mathrm{a}}{g} = \sin\alpha$ and $\frac{\mathrm{b}}{g} = \sin\beta$) the result for the grating constant g is:

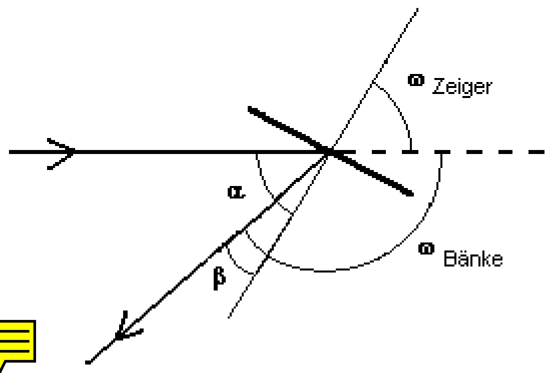
$$g = \frac{\lambda}{\sin\alpha + \sin\beta} \qquad (1)$$

The resulting number of grating lines N per mm is:

$$N = \frac{1}{1000 \cdot g} \qquad (2)$$

The angles α and β can be determined for this setup from the angle of the holographic grating with respect to the optical axis ω_{pointer} (angle of rotation) and the angle between the optical benches ω_{benches} (direction of the diffracted light beam). It can be observed for three different situations:

<u>1. The angle of incidence is larger than the angle of diffraction:</u> $\alpha > \beta$



[A1]

Fig. 2: For calculating the angle for $\alpha > \beta$

The angle of incidence α corresponds to the angle of rotation of the pillar with the holographic grating:

$$\alpha = \omega_{\text{pointer}} \qquad (3)$$

The result for the angle of diffraction β is (see fig. 2):

$$\beta = \omega_{\text{pointer}} + \omega_{\text{benches}} - 180° \qquad (4)$$

The limiting angle of observation in this situation is $\alpha = 90°$. The result for the angle of diffraction from (1) is:

$$\sin\beta = \frac{\lambda}{g} - 1 \qquad (5)$$

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2. The angle of incidence is equal to the angle of diffraction: $\alpha = \beta$

In this the so-called Littrow setup, the 1st order maximum is reflected in the direction of the incident light beam.

With equation (1) the result is

$$g = \frac{\lambda}{2 \cdot \sin\alpha} \quad (6)$$

i.e. with $\alpha = \omega_{\text{pointer}}$ (3):

$$g = \frac{\lambda}{2 \cdot \sin\omega_{\text{pointer}}} \quad (7)$$

The advantage of the Littrow setup is that the measuring error for g (or λ) is small because only one angle needs to be determined. The measuring error can be further reduced by calculating the average value of the angles for both sides. They are determined by rotating the holographic grating clockwise and counter-clockwise.

3. The angle of incidence is smaller than the diffraction angle: $\alpha < \beta$

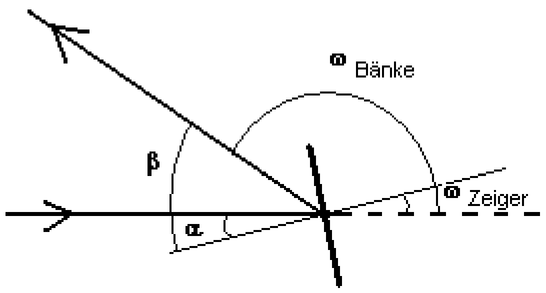

Fig. 3: For calculating the angle for $\alpha < \beta$

The result for the angle of diffraction β is (see fig. 3):

$$\beta = 180° + \omega_{\text{pointer}} - \omega_{\text{benches}} \quad (8)$$

The limiting angle of observation in this situation is $\beta = 90°$. The result for the angle of diffraction from (1) is:

$$\sin\alpha = \frac{\lambda}{g} - 1 \quad (9)$$

Apparatus

1 optical bench with standard profile, 1 m 460 32
1 optical bench with standard profile, 0.5 m 460 335
1 rail connector with circular scale.........................460 341
5 optics riders 90/50 ....460 374
1 He-Ne-laser 471 830
1 lens in frame, f = 5 mm.......................................460 01
1 projection objective..460 13
1 holographic grating, 2400 lines / mm..................471 27
1 lens in frame, *f* = 300 mm..................................460 09
1 screen 441 531

Safety notes

The He-Ne-laser conforms to the “Safety technology requirements for teaching and training equipment DIN 58126 part 6 for lasers class 2”. Provided the corresponding information in the operating instructions is observed, experimentation with the He-Ne-laser is safe.

- Do not look into the direct or the reflected laser beam.
- Avoid exceeding the dazzling limit (i.e. no observer may feel dazzled).

Setup

Fig. 4: Experimental setup

– First of all align the two optical benches with their feet at the same level and fix them using the adjustment screws.

– Screw the two rails together by means of the rail connector so that the circular angle scale is firmly attached to the rail on which the laser is mounted.

– Set up the optical elements as shown in the figure and adjust to the same height.

– Mount the laser approx. at the centre of the 1 m long optical bench and the holographic grating (in the rail connector) in such a way that the laser beam hits the centre of the holographic grating.

– Set the pointer on the pillar of the rotary joint to precisely 0° and rotate the holographic grating in the pillar so that the laser beam reflects onto itself and fix it by means of the thumbscrew. For fine adjustment use the thumbscrews on the holographic grating. (If necessary, the pillar can be fixed by means of the lower thumbscrews. Loosen them again later.)

– Place the diverging lens f = 5 mm) approx. 5 cm in front of the laser and align it in such a way that the entire holographic grating is illuminated. If necessary rotate the laser a little. The square mirror reflection should be symmetrically around the diverging lens.

– Position the projection objective approx. 15 cm in front of the diverging lens in order to obtain a parallel light beam. To do this observe the point-shaped reflection on the front of the laser (autocollimation). This reflection should hit the laser outlet opening. If necessary readjust by means of the thumbscrews on the holographic grating.

– Place the screen at the end of the optical bench 0.5 m and place the lens (*f* = 300 mm) approx. 30 cm in front of the screen.

Carrying out the experiment

a) Determination of the grating constant in the Littrow setup : $\alpha = \beta$

– Rotate the holographic grating counter-clockwise (angle +) until the 1st order maximum is reflected in the direction of the incident light beam. To do this observe the reflection on the front of the laser.

– Measure the angle of rotation ω_{pointer} of the holographic grating.

– Then rotate the holographic grating clockwise (angle -) and repeat the measurement.

b) Determination of the grating constant for $\alpha > \beta$

– First of all determine from the limiting angle the angle between the rails ω_{benches} at which the 1st order maximum will be observed with the smallest diffraction angle β .

– Rotate the optical bench 0.5 m clockwise until $\omega_{\text{benches}} = 125°$.

– Rotate the pillar with the holographic grating until the 1st order maximum hits the lens (f = 300 mm). If necessary, shift the lens until the screen is precisely located at its focal point.

– If necessary, slightly correct the angle ω_{pointer} of the pillar with the holographic grating until the 1st order maximum precisely falls on the centre of the screen.

– Read of the angle ω_{benches} between the optical benches and the angle ω_{pointer} of the holographic grating.

– Set ω_{benches} to a larger angle and repeat the experiment.

c) Determination of the grating constant for $\alpha < \beta$

– First of all determine from the limiting angle from (5) the angle between the rails ω_{benches} at which the 1st order maximum can be observed with the largest diffraction angle β .

– Rotate the optical bench 0.5 m clockwise until $\omega_{\text{benches}} = 155°$.

– Repeat the experiment as described under b).

– Set ω_{benches} to smaller angles and repeat the experiment.

Note: If there is not sufficient space the optical bench can be set up as described under b). In this case the holographic grating must correspondingly be rotated clockwise (angle + to angle -).

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Measuring examples

a) Determination of the grating constant in the Littrow setup :

– Calculation of the grating constant from the known wavelength of the He-Ne laser $\lambda = 632.8\ \mathrm{nm}$

Angle +	Angle -	Average	Grating constant from (7)	Lines per mm from (2)
$\omega_{\mathrm{pointer}}$	$\omega_{\mathrm{pointer}}$	$\overline{\omega_{\mathrm{pointer}}}$	*g*	*N*
49°	50°	49.5°	$4.16\ 10^{-7}$ m	2403

The average value found corresponds well to the specification of *N* = 2400 lines per mm.

b) Determination of the grating constant $\alpha > \beta$

– With equation (5) the limiting angle is: $\beta \approx 31°$

from this with (4) $\omega_{\mathrm{benches}} \approx -59°$ or (see fig. 2) $\omega_{\mathrm{benches}} \approx 121°$

Angle between the optical benches	Angle of the holographic grating	Diffraction angle (from 4)	Lines per mm (from 1, 2)
$\omega_{\mathrm{benches}}$	$\omega_{\mathrm{pointer}} = \alpha$	β	*N*
125°	87°	32°	2439
135°	78.5°	33.5°	2421
145°	71°	36°	2423
155°	64°	39°	2415
Average:			2425

c) Determination of the grating constant for $\alpha < \beta$

– With equation (9) the limiting angle is: $\alpha \approx 31°$

from this with (8) $\omega_{\mathrm{benches}} \approx -59°$ or (comp. fig. 3) $\omega_{\mathrm{benches}} \approx 121°$

Angle between the optical benches	Angle of the holographic grating	Diffraction angle (from 8)	Lines per mm (from 1, 2)
$\omega_{\mathrm{benches}}$	$\omega_{\mathrm{pointer}} = \alpha$	β	*N*
155°	39°	64°	2415
145°	35.5°	70.5°	2403
135°	33°	78°	2406
125°	31.5°	86.5°	2403
Average:			2407

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