Optics

Wave optics
Michelson interferometer

LD Physics Leaflets

P5.3.4.6

Determination of the separation between two spectral lines by means of the Michelson interferometer

Objects of the experiments

- Determination of the average wavelength of the yellow spectral lines for an Hg spectral lamp by means of a Michelson interferometer
- Determination of the line separation of the yellow lines by means of a Michelson interferometer

Principles

Coherence is the ability of various waves to create stationary interference effects. A temporally stationary interference structure can only be observed when the phase differences between any partial waves around a fixed point change during the observation time by less than 2π. Then the partial waves are called temporally coherent. The maximum time span Δt, during which the phase differences between all parts of the waves change by a maximum of 2π is called the coherence time Δt_C. Often the coherence length is used instead of the coherence time. This describes the distance $\Delta s_C = \frac{c}{n}\Delta t_C$ the light travels in a medium with a refractive index n during the coherence time.

In the case of two different, closely adjacent wavelengths λ_1 and λ_2 the coherent superposition of two partial waves will result in interference beating. For specific wavelength differences a strong contrast between the light and the dark rings is obtained while for other wavelength differences the contrast disappears all together. The difference in the optical wavelengths between the two intensity maxima or intensity minima which drift past a specific point on the observation screen corresponds to the median wavelength:

$$\Delta s_1 = \bar{\lambda} = \frac{\lambda_1 + \lambda_2}{2} \qquad \text{(I)}$$

The difference in the optical wavelengths between two points of maximum or minimum contrast is calculated using

$$\Delta s_2 \approx \frac{\bar{\lambda}^2}{\Delta\lambda} \qquad \text{(II)}$$

This can be used for calculating the difference in the wavelengths $\Delta\lambda$ using

$$\Delta\lambda \approx \frac{\bar{\lambda}^2}{\Delta s_2}\,. \qquad \text{(III)}$$

For sufficiently coherent sources, such as spectral lamps, even very close line separations can be determined. For this e.g. a Michelson interferometer is used. The Michelson interferometer belongs to the family of two beam interferometers. Interferometer measurements based on this interferometer type are founded on the principle described below:

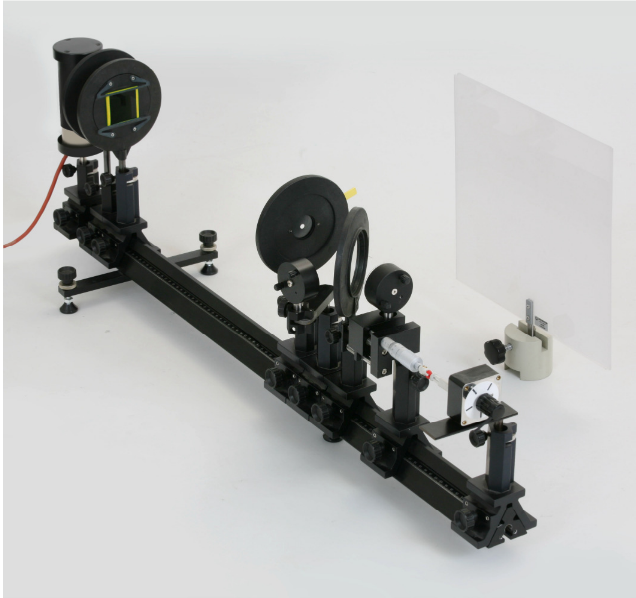

Fig. 1: Experimental setup

The coherent beam from a suitable light source is split by an optical element into two parts. The partial beams pass along different paths, are reflected back along the same path and are finally recombined and superimposed. The superimposition of the light waves creates the interference image. Because the spectral lamp emits a slightly divergent beam one does not obtain an homogeneous intensity distribution on the screen but a system made up from light and dark interference rings.

In the experiment, the mean wavelength $\bar{\lambda}$ of the yellow Hg spectral lines is determined by shifting one of the planar mirrors, by means of the fine adjustment drive, by a precise distance, which causes the optical path length of the affected partial beam to be altered. During this shift the interference lines on the observation screen drift. For the evaluation either the intensity maxima or intensity minima are counted passing a specific point on the observation screen while the planar mirror is shifted.

For the determination of the line separation $\Delta\lambda$ the distance between the two positions is determined where the interference pattern disappears, although the path length difference is less than the coherence length.

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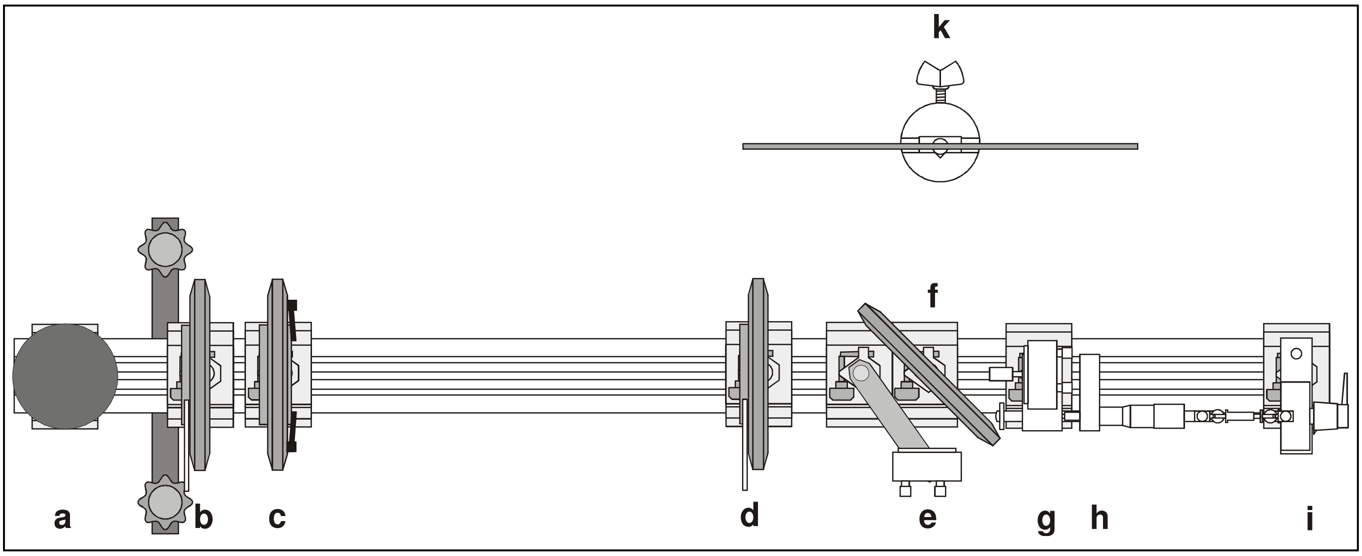

Fig. 2: The setup of the Michelson interferometer on the optical bench seen from above

a	spectral lamp Hg 100
b, d	iris diaphragm
c	580 monochromatic filter
e, g	planar mirror with fine adjustment
f	beam splitter
h	fine adjustment drive
i	reduction gears of the fine adjustment drive
k	translucent screen

Apparatus

1 spectral lamp Hg 100	451 062
1 housing for spectral lamps	451 16
1 universal choke 230 V, 50 Hz	451 30
1 optical bench with standard profile, 1 m	460 32
1 optics rider 60/50	460 373
7 optics riders 90/50	460 374
2 planar mirrors with fine adjustment	473 461
1 fine adjustment drive	473 48
1 cantilever arm	460 380
1 beam splitter	471 88
2 iris diaphragms	460 26
1 monochromatic filter, 580 nm	468 30
1 holder with spring clips	460 22
1 translucent screen	441 53
1 saddle base	300 11

Note:

The experiment can also be carried out for the double lines of an Na spectral lamp (451 111).

Safety notes:

- Only connect the spectral lamp (451 062) in its housing (451 16) to the mains via the universal choke (451 30).
- Between the light opening and the optical element (e.g. diaphragm, lens) a minimum distance of 3 cm has to be adhered to in order to prevent overheating.

Setup

Note: Carry out the measurements in a room as completely dark as possible.

Optical components with damaged or dirty surfaces can lead to errors in the interference pattern.

Treat the planar mirror and the beam splitter with great care, store them protected from dust and never touch with bare hands.

A experimental setup applying the cross connector (460 342) is shown at the end of the leaflet.

The experimental setup is shown in figure 2. For setting up, the steps described below are required:

Installation on the optical bench:

- Mount the Hg 100 spectral lamp (**a**) in the optics rider 60/50 at one end of the optical bench.
- Mount the reduction gears for the fine adjustment drive (**i**) with the magnetic base on the gear base and mount it on the other end of the optical bench.
- Clamp a planar mirror (**g**) at the top end of the fine adjustment drive (**h**) and mount it in front of the reduction gears (**i**) on the bench.
- Carefully clamp the universal coupling in the joint head of the micrometer screw of the fine adjustment drive (**h**)
- Shift the optics rider with the fine adjustment drive (**h**) and the height of the gear base of the reduction gears (**i**) in such a way that the coupling rods are neither fully stretched out nor compressed. Otherwise the measurement might become falsified because of shifting of the fine adjustment drive.
- Keep the angle between the individual elements of the couplings as small as possible (and under no circumstance larger than 45°)
- Mount one of the iris diaphragms (**b**) approx. 5 cm behind the spectral lamp Hg 100, the second iris diaphragm (**d**) approx. 25 cm in front of the planar mirror, in such a way that the centres of the diaphragms are at the same height.
- Connect the universal choke to the spectral lamp and switch on; wait for several minutes for the lamp to warm up.

Adjustment of the planar mirror (g)

After the adjustment of the planar mirror (**g**), it should reflect the light from the spectral lamp back onto its own path. Only in this case will shifting of the planar mirror (**g**) by means of the fine adjustment drive not lead to a beam movement.

- Close the iris diaphragms (**b, d**) as far as possible.
- Align the planar mirror (**g**) by adjusting the screws on the back so that the reflected beam hits both iris diaphragm (**d**) and iris diaphragm (**b**) centrally.

After this the adjustment screws on the planar mirror (**g**) must not be touched again! A change to the adjustment of the planar mirror (**g**) would mean that any shifting of the planar mirror (**g**) with the fine adjustment drive would result in a movement of the reflected beam and therefore the partial beams would no longer superimpose properly.

Beam splitter (f) and the second planar mirror (e)

- Place the beam splitter (**f**) as close as possible to the front of the planar mirror (**g**) so that the complete travel of the fine adjustment drive can be utilised. The mirror side of the beam splitter points in the direction of the spectral lamp.
- Turn the beam splitter (**f**) in such a way that the beam reflected from the planar mirror (**g**) is deflected by 90°.
- Fix the translucent screen (**k**) in the base and place it next to the optical bench so that it is hit at its centre.
- Then clamp the second planar mirror (**e**) in the cantilever arm and mount it on the optical bench in such a way that the planar mirror is hit at its centre by the partial beam reflected from the beam splitter. The distances the planar mirror (**e**) and planar mirror (**g**) are from the beam splitter (**f**) should be approximately equal. On account of the relatively small coherence length of the spectral lamp of a few millimetres, the optical wavelength of the two interferometer arms may be only very slightly different. If necessary adjust the positions of the planar mirror (**e**) and/or the beam splitter (**f**).

Superimposition of the partial beams

For the superimposition adjust only the planar mirror (**e**) (on the cantilever arm)!

- Align the planar mirror (**e**) by adjusting the screws on the back so that the beam is nearly reflected along its own path and after transmission through the beam splitter combines with the first partial beam.
- By fine adjustment of the planar mirror (**e**) the beams of the two interferometer arms can be fully superimposed. For doing this it is useful to cover the beam reflected from the planar mirror (**e**) directly in front of the planar mirror (**e**) partially with a piece of firm paper (e.g. a calling card). The position of the partial beam allowed to pass can now easily be compared to the position of the beam reflected by the planar mirror (**g**).
- Fully open iris diaphragm (**d**).

Now interference lines should become visible on the screen.

- Carefully adjust the planar mirror (**e**) in such a way that on the screen a system of concentric rings becomes visible in the centre of the lit area.
- Open the iris diaphragm (**b**) sufficiently that the contrast in the interference pattern is not affected.

Carrying out the experiment

During the experiment:

- Avoid mechanical vibration of the optical bench (e.g. do not wobble the table).
- Avoid air flow through the setup (because of flow marks) e.g. from draughts.
- Mark a location on the transparent screen (**k**) where the drifting interference lines can be counted.
- Because of the play in the gearbox, adjust the gearbox knob slowly and uniformly by gently placing the finger onto the lever of the reduction gears (**i**) and in this way until, if necessary with more turns, the interference lines start moving.
- Then give the gearbox knob at least one further turn before starting with the measurement.

Note: if the planar mirror and therefore the interference pattern moves jerkily, the slide bush of the fine adjustment drive needs to be lubricated.

Determination of the mean wavelength of the yellow spectral lines of an Hg spectral lamp

- Clamp the 580 nm filter in the holder with spring clips and mount it behind the iris diaphragm (**b**) on the optical bench
- Turn the gear knob until an area of high contrast of the interference lines has been reached. Continue rotating the gearbox knob and at the same time count the interference lines (approx. 100) drifting past the mark and the turns of the reduction gears.

Determination of the separation of the yellow spectral lines of an Hg spectral lamp

- Continue rotating the gearbox knob until the interference pattern is only just visible.
- Continue rotating the gearbox knob to make the interference pattern reappear. At the same time count the rotations of the reduction gears until the interference pattern has disappeared once again.

Measuring example and evaluation

Determination of the mean wavelength of the yellow spectral lines of an Hg spectral lamp

The number N of rotations of the reduction gears, the total shift Δx_1 of the planar mirror, the mean wavelength $\bar{\lambda}$ of the spectral lines and the number of rotations Z of the intensity maxima are given by the relationship below:

$Z \cdot \bar{\lambda} = \Delta s_1 = 2 \cdot \Delta x_1$ with $\Delta x_1 = N \cdot 5\ \mu\text{m}$

The factor 2 in this equation takes into account that the geometric path is changed for both the direct and the reflected beam by Δx_1.

For $\bar{\lambda}$ also the equation applies

$$\bar{\lambda} = 2 \cdot \frac{\Delta x_1}{Z} = 2 \cdot \frac{N \cdot 5\mu\text{m}}{Z}$$

In the example, the number of intensity maxima counted was $Z = 52$ and the number of rotations was $N = 3$. From this the wavelength of the green Hg spectral line of the Hg spectral lamp is determined to be:

$\bar{\lambda} = 577$ nm.

The values found in the literature are $\lambda_{1,theo} = 576.96$ nm and $\lambda_{2,theo} = 579.07$ nm and therefore $\bar{\lambda}_{theo} = 578.02$ nm.

Determination of the separation of the yellow spectral lines of an Hg spectral lamp

The difference in the lengths of the optical paths between two points of maximum or minimum contrast is calculated according to formula (II) as

$$\Delta s_2 = 2 \cdot \Delta x_2 \approx \frac{\bar{\lambda}^2}{\Delta\lambda},$$

with Δx_2 being the total shift of the planar mirror. By means of equation (I) this allows the difference of the wavelengths $\Delta\lambda$ to be determined using

$$\Delta\lambda \approx \frac{\bar{\lambda}^2}{2\Delta x_2}.$$

In the experiment the distance between two points of minimum contrast is 17 turns of the gearbox knob, i.e. $\Delta x_2 = 85$ µm. This gives the result

$\bar{\lambda} = 577$ nm: $\Delta\lambda = 2$ nm.

The values found in literature of $\lambda_{1,theo} = 576.96$ nm and $\lambda_{2,theo} = 579.07$ nm result in $\Delta\lambda_{theo} = 2.11$ nm.

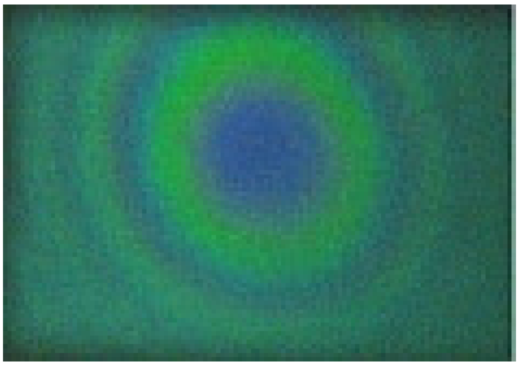

Fig. 3: Interference pattern on the screen without filter

Discussion

By means of the interferometric measurement the separation between the yellow spectral lines of the Hg spectral lamp can be determined precisely. The measuring precision is higher the larger the total shift Δs_1 and therefore the number of turns and the number of counted intensity maxima. The precision of the line separation depends on the one hand on the relative (!) precision of the mean wavelength and the value of the total shift Δs_2 and therefore also on the number of turns.

The smaller the separation between the lines, the larger the total shift Δs_2 between two points of minimum or maximum contrast. This allows a more precise determination. If Δs_2 is however larger than the coherence length, a purely manual evaluation is no longer possible.

Notes:

If the intensity change at one point of the interference image is electronically recorded (e.g. by means of a photodiode) as a function of the total shift Δs this allows, by means of a Fourier transformation, all of the directly involved frequencies/wavelengths to be determined, i.e. the entire spectrum. This method is in general call Fourier transformation spectroscopy and is widely practised e.g. as *Fourier transform infrared spectroscopy* (FTIR).

Alternative Setup with cross connector (460 342)

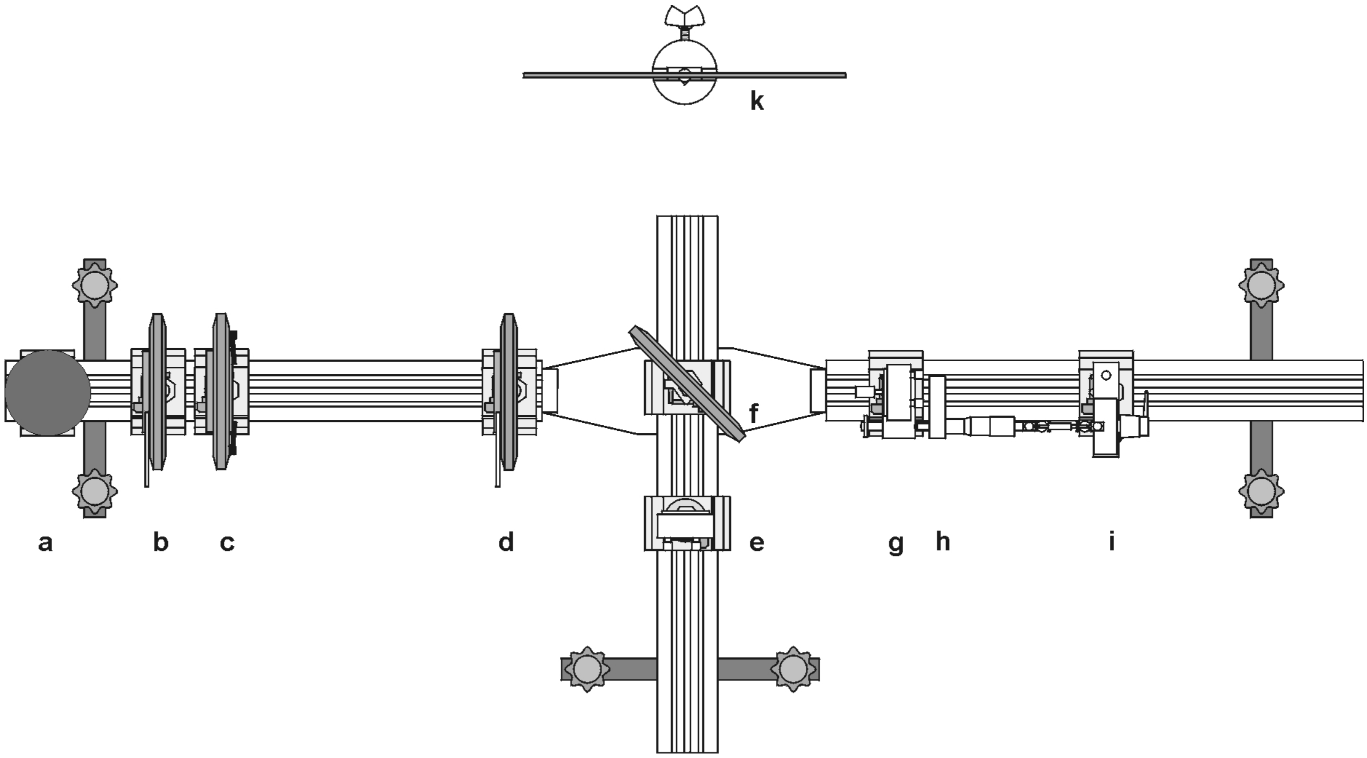

Fig. 4: The setup of the Michelson interferometer on the optical bench seen from above

a	spectral lamp Hg 100
b, d	iris diaphragm
c	580 monochromatic filter
e	planar mirror with fine adjustment in extension rod
f	beam splitter
g	planar mirror with fine adjustment
h	fine adjustment drive
i	reduction gears of the fine adjustment drive
k	translucent screen

Connect the optical benches with the cross connector as shown in Fig. 4. Then, the setup of the optical components is similar to the setup on a single optical bench.

Apparatus

1 spectral lamp Hg 100	451 062
1 housing for spectral lamps	451 16
1 universal choke 230 V, 50 Hz	451 30
3 optical bench with standard profile, 0.5 m	460 335
1 cross connector	460 342
1 optics rider 60/50	460 373
7 optics riders 90/50	460 374
1 extension rod	460 385
2 planar mirrors with fine adjustment	473 461
1 fine adjustment drive	473 48
1 beam splitter	471 88
2 iris diaphragms	460 26
1 monochromatic filter, 580 nm	468 30
1 holder with spring clips	460 22
1 translucent screen	441 53
1 saddle base	300 11