Diffraction at one- and two-dimensional gratings

Objects of the experiment

- Investigating the diffraction phenomena at groove gratings and crossed gratings.
- Determining the wavelength.
- Determining the grating constant.

Principles

The nature of light was a controversial issue for a long time. In 1690, Christiaan Huygens interpreted light as a wave phenomenon; in 1704, Isaac Newton described the light beam as a current of particles. This contradiction was resolved by quantum mechanics, and the idea of wave-particle duality came up. The experiment on diffraction of light at a double slit is a crucial experiment for evidencing the wave character of light, and it is found again among the introductory experiments into quantum mechanics.

Diffraction phenomena always occur when the free propagation of light is changed by obstacles such as iris diaphragms or slits. The deviation from the rectilinear propagation of light observed in this case is called diffraction.

When diffraction phenomena are studied, two types of experimental procedure are distinguished:

In the case of *Fraunhofer diffraction*, parallel wave fronts of the light are studied in front of the diffraction object and behind it. This corresponds to a light source which is at infinite distance from the diffraction object on one side and, on the other side, a screen which, too, is at infinite distance from the diffraction object.

In the case of *Fresnel diffraction*, the light source and the screen are at a finite distance from the diffraction object. With increasing distances, the Fresnel diffraction patterns are increasingly similar to the Fraunhofer patterns.

With the coherent light of a laser, it is very convenient to investigate the diffraction properties of groove gratings with different grating constants. Due to the diffraction of the incoming parallel light at the slit apertures, the light propagates into the geometrical shadow area. The presence of light in the shadow area cannot be explained by the laws of geometrical optics. The pattern of bright and dark spots observed on the screen can be explained with reference to diffraction and interference phenomena if the properties of waves are attributed to the light.

Fig. 1 suggests a simple approach to make it plausible that constructive interference always occurs on the screen at positions where the path difference Δs_k of equivalent partial

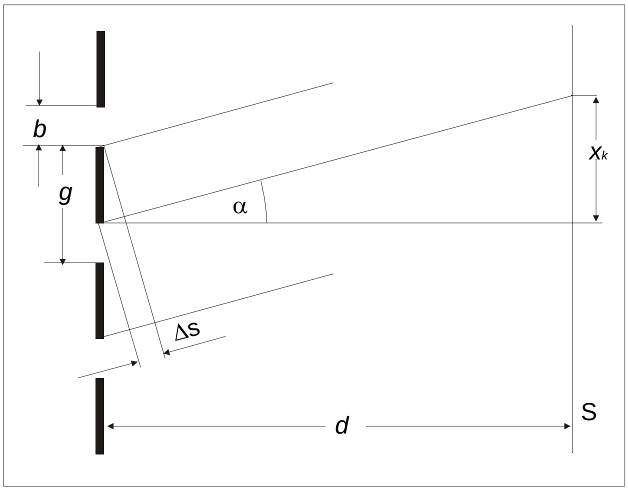

Fig. 1: Geometrical relations in the diffraction of light at a groove grating
b: slit width
g: slit spacing
d: distance between the screen and the slit
x_k: distance of the k-th intensity maximum from the centre
α: direction in which the k-th constructive interference is observed
Δs: path difference
S: screen

bundles from neighbouring slit apertures is an integer multiple of the wavelength:

$$\Delta s_k = k \cdot \lambda \qquad k = 1, 2, 3, \ldots \qquad \text{(I)}$$

Let us assume that these locations are given at distances x_k from the bright centre of the diffraction pattern. If the screen distance *d* is great as compared with the slit spacing *g*, we have

$$\frac{\Delta s_k}{g} \approx \alpha_k \approx \frac{x_k}{d} \qquad \text{(II)}$$

LD Didactic GmbH · Leyboldstrasse 1 · D-50354 Huerth / Germany · Phone: (02233) 604-0 · Fax: (02233) 604-222 · e-mail: info@ld-didactic.de

for small diffraction angles α. Thus the wavelength λ can be determined by measuring the distances x_k:

$$k \cdot \frac{\lambda}{g} = \frac{x_k}{d} \quad \text{or} \quad \lambda = \frac{x_k}{k} \cdot \frac{g}{d} \qquad \text{(III)}.$$

For large diffraction angles α, x_k is bounded by the approximation $\sin\alpha \approx \tan\alpha$.

In an arrangement of N slits of width b and spacing g, the intensity distribution on the screen is given by:

$$I(\alpha) \sim \frac{\sin^2(\frac{\phi}{2})}{(\frac{\phi}{2})^2} \cdot \frac{\sin^2(\frac{N}{2}\varphi)}{\sin^2(\frac{\varphi}{2})} \qquad \text{(IV)}$$

with

$$\phi = \frac{2\pi}{\lambda} b \sin(\alpha) \quad \text{and} \quad \varphi = \frac{2\pi}{\lambda} g \sin(\alpha)$$

The second factor on the right-hand side of equation (IV) describes a periodic sequence of intensity maxima and intensity minima which would be observed in the case of diffraction at an arrangement of N equally spaced, infinitely narrow slits.

The first factor on the right-hand side of equation (IV) describes the influence of the finite slit width b. This factor corresponds to the "envelope" of the diffraction pattern and is equal to the diffraction function of a single slit of width b. Thus the diffraction pattern of multiple slits ($N \geq 2$) is modulated by the diffraction pattern of a single slit.

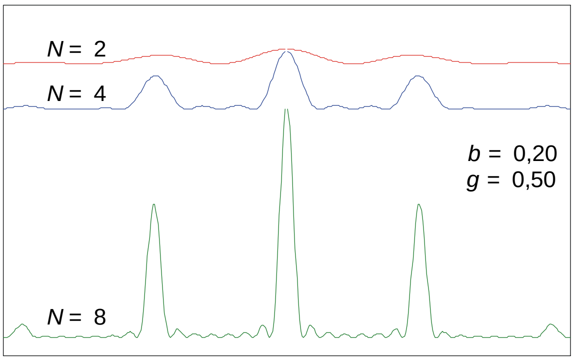

Fig. 2: Formation of the diffraction pattern of gratings starting from a double slit and then with increasing line number N. The intensity distributions shown have been calculated using equation (IV).

Starting from the double slit (N = 2), Fig. 2 shows how a diffraction pattern of a grating is formed. Here the slit width b and the slit spacing g have not been varied. It is seen that the intensity of the secondary maxima fades whereas that of the principal maxima becomes stronger and stronger.

***Remark:** the study of diffraction patterns of a double slit and of multiple slits in dependence on the slit width b and the slit spacing g is described in the experiment P5.3.1.2.*

The superposition of two groove gratings at an angle of 90° results in a crossed grating. The diffraction pattern can also be described with equation (III), whereby the relation has now to be applied for either of the two perpendicular components according to the superposition principle.

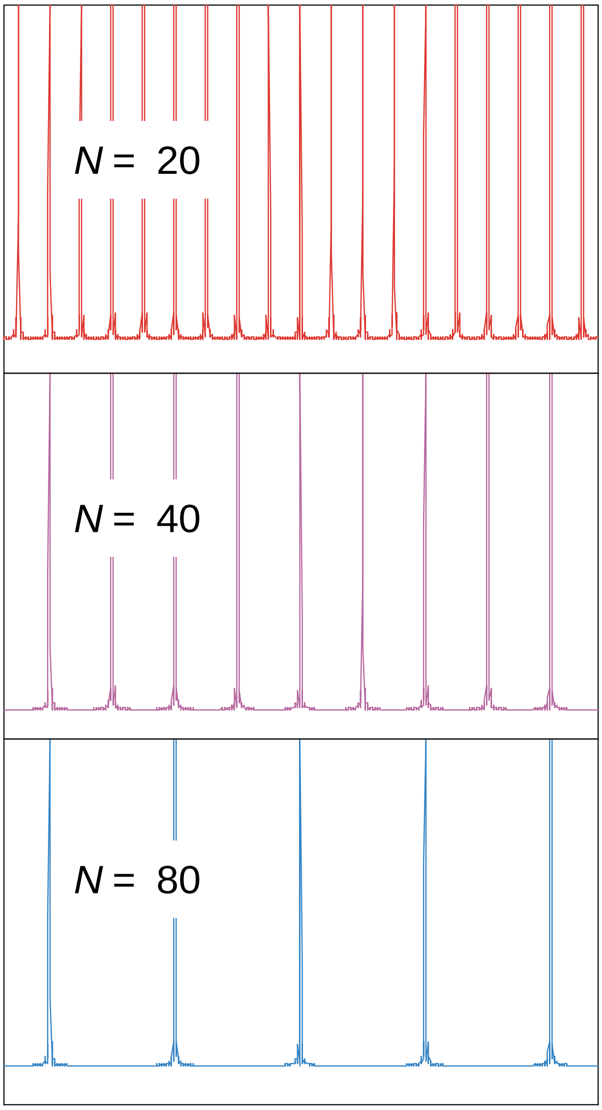

Fig. 3: Diffraction patterns of groove gratings with different grating constants g (different line numbers/cm). The intensity distributions shown have been calculated for the diaphragm with three gratings (469 87) using equation (IV).

Apparatus

1 Diaphragm with 3 gratings	469 87
1 Diaphragm with 2 wire-mesh gratings	469 88
1 Holder with spring clips	460 22
1 He-Ne-Laser, linear polarized	471 830
1 Lens in frame f = +5 mm	460 01
1 Lens in frame f = +50 mm	460 02
1 Optical bench, standard cross section 1 m	460 32
4 Optics rider 60/34	460 370
1 Translucent screen	441 53
1 Saddle base	300 11

LD Didactic GmbH · Leyboldstrasse 1 · D-50354 Huerth / Germany · Phone: (02233) 604-0 · Fax: (02233) 604-222 · e-mail: info@ld-didactic.de

Safety notes

The He-Ne laser meets the requirements according to class 2 of EN 60825-1 “Safety of laser equipment”. If the corresponding notes of the instruction sheet are observed, experimenting with the He-Ne laser is safe.

- Never look into the direct or reflected laser beam.
- No observer must feel dazzled.

Setup

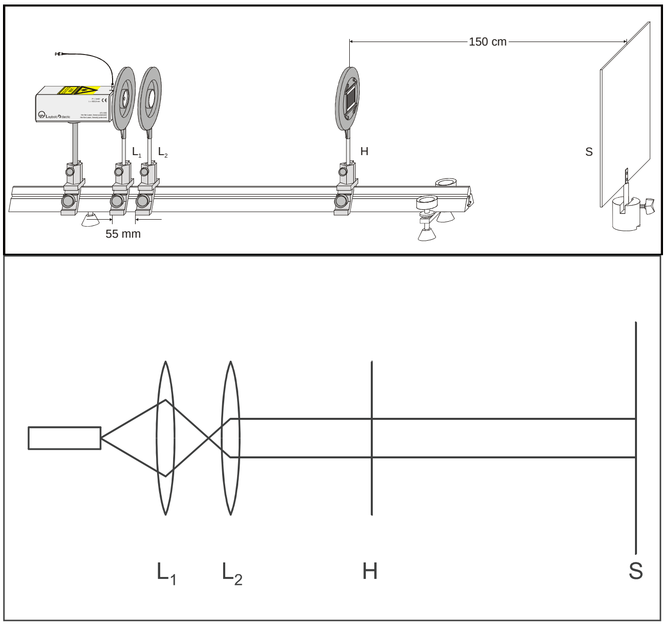

Fig. 4: Experimental setup for observing diffraction at a grating
L_1: lens *f* = +5 mm
L_2: lens *f* = +50 mm
H: holder for diffraction objects
S: screen

The experimental setup is illustrated in Fig. 4a. Fig. 4b is a schematic representation of the ray path. At first the spherical lens L_1 with the focal length *f* = +5 mm expands the laser beam. The subsequent converging lens L_2 with the focal length *f* = +50 mm is so positioned that its focus lies somewhat below the focus of the spherical lens. This leads to the laser beam being slightly expanded and running approximately parallel along the optical axis (Fraunhofer’s point of view).

Remark: *the adjustment is easier in a slightly darkened room.*

- Using a rider, mount the He-Ne laser to the optical bench as shown in Fig. 4.
- Set up the screen S at a distance of approx. 1.90 m from the laser.
- Direct the laser towards the screen, and switch it on.
- Place the holder for diffraction objects H on the optical bench at a distance of approx. 50 cm from the laser with the diaphragm with 3 gratings being clamped. Adjust the height of the laser so that the laser beam passes the middle of the diaphragm.
- Place the spherical lens L_1 with the focal length f = +5 mm in front of the laser at a distance of approx. 1 cm. (The laser light has to cover the diaphragm.)
- Remove the holder H for diffraction objects. Position the converging lens L_2 with the focal length f = +50 mm in front of the spherical lens L_1 at a distance of approx. 55 mm, and displace it along the optical bench towards the spherical lens L_1 until the laser beam is imaged sharply on the screen.
- Displace the converging lens L_2 on the optical bench somewhat further towards the spherical lens L_1 until the diameter of the laser beam on the screen is approx. 6 mm. (Now the laser beam should have a constant circular profile along the optical axis.)

Remark: *in order to check whether the beam diameter of the laser is constant between the lens and the screen, hold a sheet of paper in the ray path and follow the profile of the laser beam along the optical axis.*

- Put the holder for diffraction objects back into the ray path and displace it until the distance between the screen and the diffraction object is 1.50 m. (If necessary, displace the lens L_2 slightly until the diffraction pattern is imaged sharply.)

Carrying out the experiment

a) Diffraction pattern of a grating in dependence on the slit spacing

- Investigate the diffraction patterns of the groove gratings in the diaphragm with 3 gratings (469 87) in dependence on the slit spacing *g*.
  For this observe the diffraction patterns of the gratings with the slit spacings 0.185 mm (80 lines/cm), 0.25 mm (40 lines/cm) and 0.50 mm (20 lines/cm) one after another.

Remark: *pay attention to the fact that the change from g = 0.185 to g = 0.25 and then to g = 0.50 mm always leads to an additional maximum between the maxima of the respective previous diffraction pattern.*

- For determining the distances x_k between neighbouring maxima hold a sheet of paper on the screen, and, using a soft pencil, mark the locations of the intensity maxima (bright spots).

b) Diffraction pattern of a crossed grating

- Observe the diffraction patterns of two mutually complementary crossed gratings – grid of points and bar grating (469 88).
- For determining the distances x_k between neighbouring maxima hold a sheet of paper on the screen, and, using a soft pencil, mark the locations of the intensity maxima (bright spots).

Remark: *pay attention to the fact that the distances x_k in the vertical and horizontal direction are equal and correspond to those of experiment a).*

Measuring example

a) Diffraction pattern of a grating in dependence on the slit spacing

Table 1: distances x_k

Experiment a): groove grating		
$\frac{g}{mm}$	$\frac{x}{mm}$	$\frac{g}{mm}$
0.50	1.91	0.50
0.25	3.79	0.25
0.125	7.38	0.13

b) Diffraction pattern of a crossed grating

Table 2: distances x_k

Grid of points		
$\frac{g}{mm}$	$\frac{x}{mm}$	$\frac{\lambda}{mm}$
0.25 (vertical)	3.80	633
0.25 (horizontal)	3.79	631
Bar grating		
$\frac{g}{mm}$	$\frac{x}{mm}$	$\frac{\lambda}{mm}$
0.25 (vertical)	3.78	629
0.25 (horizontal)	3.79	631

Evaluation

a) Diffraction pattern of a grating in dependence on the slit spacing

Measure the distances x_k between the intensity maxima.

Using the wavelength of the He-Ne laser λ = 633 nm and equation (III) in the form

$$g = \lambda \cdot d \cdot \frac{k}{x_k} \qquad \text{(V)},$$

determine the slit spacing g.

Table 1: distances x_k

Experiment a): groove grating		
$\frac{g}{mm}$	$\frac{x}{mm}$	$\frac{g}{mm}$
0.50	1.91	0.50
0.25	3.79	0.25
0.125	7.38	0.13

b) Diffraction pattern of a crossed grating

Measure the distances x_k between the intensity maxima and, using equation (III) determine the wavelength λ of the He-Ne laser.

Table 2: distances x_k

Grid of points		
$\frac{g}{mm}$	$\frac{x}{mm}$	$\frac{\lambda}{mm}$
0.25 (vertical)	3.80	633
0.25 (horizontal)	3.79	631
Bar grating		
$\frac{g}{mm}$	$\frac{x}{mm}$	$\frac{\lambda}{mm}$
0.25 (vertical)	3.78	629
0.25 (horizontal)	3.79	631

Mean value: λ = 631 nm

Results

Gratings are multiple slits where the diffraction pattern consists of the intensity of the principal maxima only. The greater the number of diffracting apertures, the more intensive are the intensity maxima. The distance between neighbouring intensity maxima of the grating's diffraction pattern is proportional to the line number of the grating. The distances between the intensity maxima increase with increasing line number.

Supplementary information

Gratings with a great grating constant g can be used for an exact determination of the wavelength.

Moreover, gratings are widely used as an aid in spectral dispersion of light (electromagnetic radiation) in the spectral range from the ultraviolet region until far into the infrared region. In contrast to prisms, the deflection of red light is strongest at a grating.

J. Fraunhofer (1821) was the first to manufacture optical groove gratings by scratching parallel, closely spaced grooves into a glass plate using diamonds of a cutting machine. The first high-quality gratings for spectroscopic purposes were manufactured by *H. A. Rowland* (1882). Nowadays gratings are produced with the aid of interferometrically controlled cutting machines.