Determining inductive reactance with a Maxwell measuring bridge

Objects of the experiments

- Determining the inductance and the ohmic resistance of air coils as functions of the number of turns by adjusting a Maxwell measuring bridge.
- Demonstrating that the balance condition is independent of the frequency of the AC voltage.
- Comparing the measuring values with the values calculated from the geometrical data of the coils.

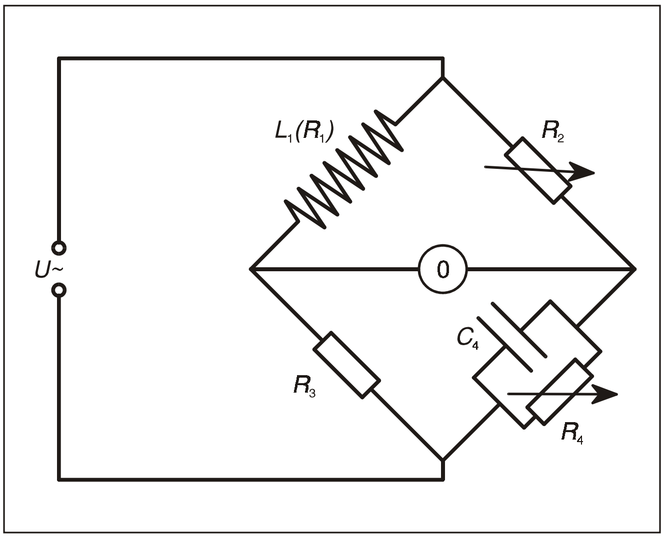
Principles

The Wheatstone measuring bridge is used to determine ohmic resistance in DC and AC circuits. In an analogue bridge circuit, the Maxwell measuring bridge (see Fig. 1), inductive reactance can be determined. This measuring bridge, too, consists of four passive bridge arms, which are connected to one another in a square, an indicator arm with a balance indicator and a supply arm with the voltage source. The current in the indicator arm is made zero by adjusting variable elements in the bridge arm. Then the involved complex reactances fulfil the fundamental balance condition

$$Z_1 = Z_2 \cdot \frac{Z_3}{Z_4} \quad \text{(I)},$$

from which the quantity to be measured

Fig. 1 Diagram of a Maxwell measuring bridge for determining an inductive reactance Z_1



$$Z_1 = R_1 + i \cdot 2\pi \cdot f \cdot L_1 \quad \text{(II)}$$

L_1: inductance, R_1: ohmic resistance,
f: frequency of the applied AC voltage

can be determined.

As the ohmic portion of Z_1 is also to be balanced, this circuit is somewhat more complicated than the Wheatstone measuring bridge: Z_2 is a variable ohmic resistance, Z_3 is a fixed ohmic resistance, and Z_4 is a parallel connection of a capacitive reactance and a variable ohmic resistance as a reference impedance.

Therefore,

$$Z_2 = R_2 \text{ and } Z_3 = R_3 \quad \text{(III)}$$

and

$$\frac{1}{Z_4} = \frac{1}{R_4} + i \cdot 2\pi \cdot f \cdot C_4 \quad \text{(IV)}.$$

In the case of zero balance, the purely ohmic portion of Z_1 is

$$R_1 = R_2 \cdot R_3 \cdot \frac{1}{R_4} \quad \text{(V)}$$

and the purely inductive portion is

$$L_1 = R_2 \cdot R_3 \cdot C_4 \quad \text{(VI)},$$

regardless of the frequency f.

In the experiment, the inductances and the ohmic resistances of two air coils with equal dimensions, but different numbers of turn are determined and compared with one another. An earphone, an oscilloscope or a Sensor-CASSY can be used as a balance indicator.

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Apparatus

1 coil, 500 turns	590 83
1 coil, 1000 turns	590 84
1 plug-in board, A4	576 74
1 resistor 100 Ω, 0.5 W, STE 2/19	577 01
2 potentiometer 1 kΩ, 2 W, STE 4/50, 10-turn	57793
1 capacitor 4.7 µF, 63 V, STE 2/19	578 16
2 sets of 10 bridging plugs	501 48
1 function generator S 12, 0.1 Hz-20 kHz	522 621
Connection leads	
1 earphone 2 kΩ	579 29
or	
1 two-channel oscilloscope 303	575 211
1 screened cable BNC/4 mm	575 24
or	
1 Sensor-CASSY	524 010
1 CASSY Lab	524 200

The inductance of an air coil is given by

$$L = \mu_0 \cdot N^2 \cdot k \cdot \frac{a^2}{d} \qquad \text{(VII)}$$

$\mu_0 = 4\pi \cdot 10^{-7} \frac{\text{Vs}}{\text{Am}}$: magnetic constant,

N: number of turns, d = 3.3 cm: coil length, a = 2.5 cm: mean edge length of the quadratic cross section
k = 0.7: demagnetization factor, which takes into account the finite coil length

The ohmic resistance of the wire loops is

$$R = \rho \cdot \frac{s}{q} = \rho \cdot \frac{N \cdot 4 \cdot a}{q} \qquad \text{(VIII)}$$

$\rho = 0{,}0178 \frac{\Omega \cdot \text{mm}^2}{\text{m}}$: resistivity of copper

q: wire cross section, $s = 4 \cdot N \cdot a$: wire length

Setup

The experimental setup is illustrated in Fig. 2.

– Connect the function generator as an AC voltage source, and set the maximum output voltage and the signal shape ∿.

– Connect the earphone, the oscilloscope or the Sensor-CASSY between the connection points as a balance indicator.

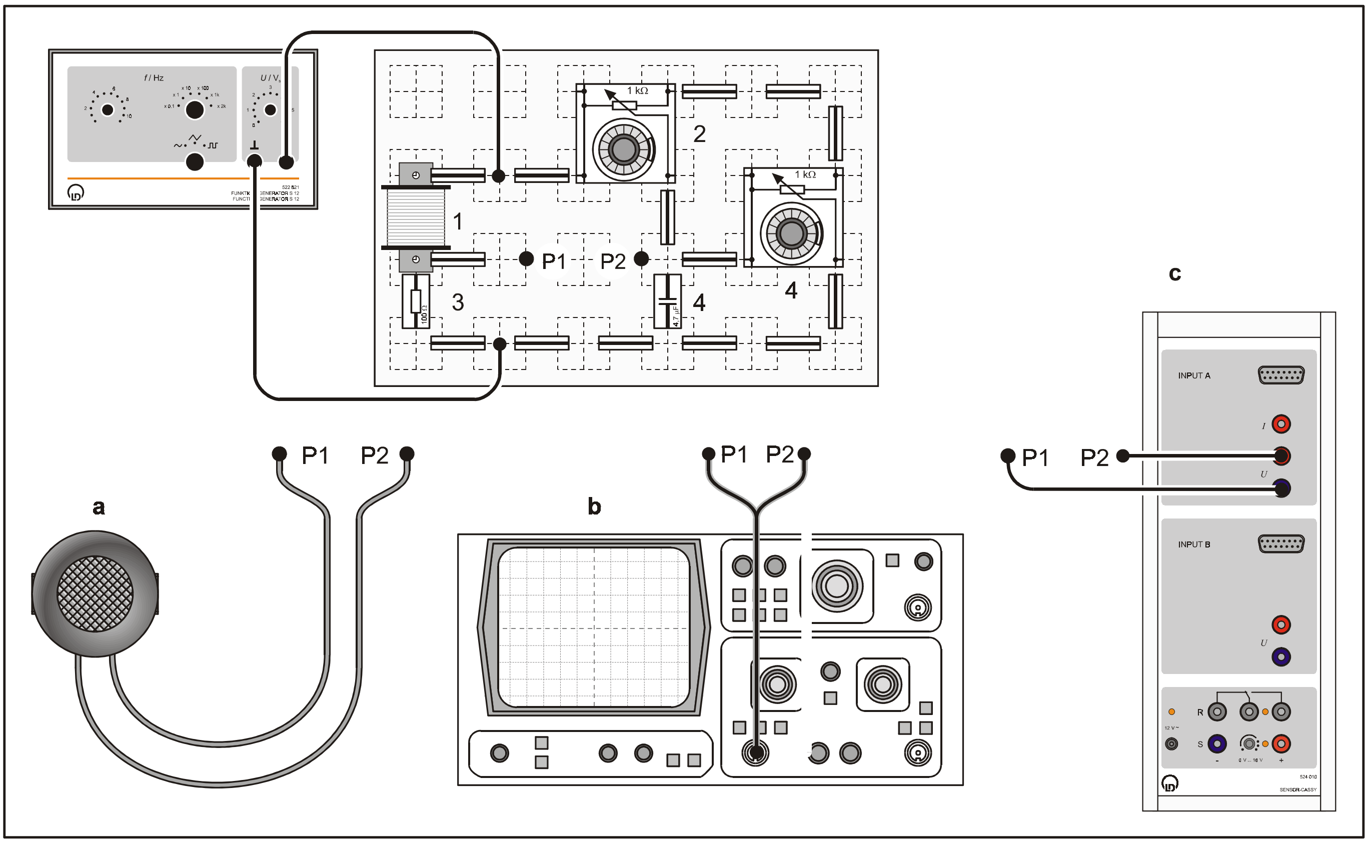

Fig. 2 Experimental setup for determining inductive reactance by means of a Maxwell measuring bridge

Carrying out the experiment

Remark concerning the selection of the frequency of the AC voltage:

If the Sensor-CASSY is used as a balance indicator, the frequency f should not exceed 500 Hz because otherwise the r.m.s. value is not determined correctly. If the earphone is used, higher frequencies are recommendable in order to ensure sufficient aural sensitivity.

Oscilloscope settings:

Coupling: AC

Deflection: 10 mV/DIV.

Trigger: AC

Time base: 5 ms/DIV. (f = 100-500 Hz)

Sensor-CASSY settings:

Sensor input settings A1:

Measurement quantity: U_{A1}, r.m.s. values, measuring range: 0 V... 0.21 V

Measuring parameters:

automatic recording, repeating measurement,

Trigger: U_{A1} 0.0000 V rising

Interval: 1 ms (f = 50 Hz), 500 µs (f = 100 Hz), 200 ms (f = 250 Hz, 100 µs (f = 500 Hz)

Number: 1000

a) Coil with 1000 turns

– Insert the coil with 1000 turns.
– Switch the function generator on by connecting the plug-in power supply.
– Set a frequency that fits the balance indicator used.
– Alternately vary the resistances R_2 and R_4 until the signal at the balance indicator is minimal (zero).
– Vary the frequency in the minimum to check the balance.

b) Coil with 500 turns

– Insert the coil with 500 turns.
– Repeat the measurement.

Measuring example

Fixed parameters: C_4 = 4.7 µF, R_3 = 100 Ω

a) Coil with 1000 turns

R_2: scale value 0.346

R_4: scale value 1.970

Balance checked for f = 50, 100, 200 and 500 Hz

b) Coil with 500 turns,

R_2: scale value 0.090

R_4: scale value 2.360

Balance checked for f = 50, 100, 200 and 500 Hz

Evaluation

a) Coil with 1000 turns

$$R_2 = \frac{0{,}346}{10}\,\mathrm{k\Omega} = 34{,}6\,\Omega$$

$$R_4 = \frac{1{,}970}{10}\,\mathrm{k\Omega} = 197{,}0\,\Omega$$

After inserting these values in Eqs. (V) and (VI), we obtain:

L_1 = 16.3 mH, R_1 = 17.6 Ω

From the dimensions of the coils and with the aid of Eqs. (VII) and (VIII), we calculate:

L_1 = 16.4 mH, R_1 = 17.5 Ω (wire diameter: 0.36 mm)

b) Coil with 500 turns

$$R_2 = \frac{0{,}090}{10}\,\mathrm{k\Omega} = 9{,}0\,\Omega$$

$$R_4 = \frac{2{,}360}{10}\,\mathrm{k\Omega} = 236{,}0\,\Omega$$

After inserting these values in Eqs. (V) and (VI), we obtain:

L_1 = 4.2 mH, R_1 = 3.8 Ω

From the dimensions of the coils and with the aid of Eqs. (VII) and (VIII), we calculate:

L_1 = 4.1 mH, R_1 = 3.7 Ω (wire diameter: 0.55 mm)

Result

With the aid of a Maxwell measuring bridge, the capacitance of a capacitor can be determined. The balance parameters are independent of the frequency of the applied AC voltage.

The inductance of an air coil is proportional to the square of the number of turns.