Determining the impedance in circuits with coils and ohmic resistors

Objects of the experiment

- Determining the total impedance and the phase shift in a series connection of a coil and a resistor.
- Determining the total impedance and the phase shift in a parallel connection of a coil and a resistor.

Principles

If an alternating voltage

$$U = U_0 \cdot \cos(\omega \cdot t) \text{ with } \omega = 2\pi \cdot f \quad \text{(I)},$$

is applied to a coil with the inductance L, the current flowing through the coil is

$$I = \frac{U_0}{\omega \cdot L} \cdot \cos\left(\omega \cdot t - \frac{\pi}{2}\right) \quad \text{(II)}.$$

Therefore an inductive reactance

$$X_L = \omega \cdot L \quad \text{(III)}$$

is assigned to the coil, and the voltage is said to be phase-shifted with respect to the current by 90° (see Fig. 1). The phase shift is often represented in a vector diagram.

Series connection

If the coil in connected in series with an ohmic resistor, the same current flows through both components. This current can be written in the form

$$I = I_0 \cdot \cos(\omega \cdot t - \varphi_S) \quad \text{(IV)}$$

where φ_S is unknown for the time being. Correspondingly, the voltage drop is

$$U_R = R \cdot I_0 \cdot \cos(\omega \cdot t - \varphi_S) \quad \text{(V)}$$

at the resistor and

$$U_L = X_L \cdot I_0 \cdot \cos\left(\omega \cdot t - \varphi_S + \frac{\pi}{2}\right) \quad \text{(VI)}$$

at the coil. The sum of these two voltages is

$$U_S = \sqrt{R^2 + X_L^2} \cdot I_0 \cdot \cos(\omega \cdot t) \quad \text{(VII)}$$

if φ_S fulfils the condition

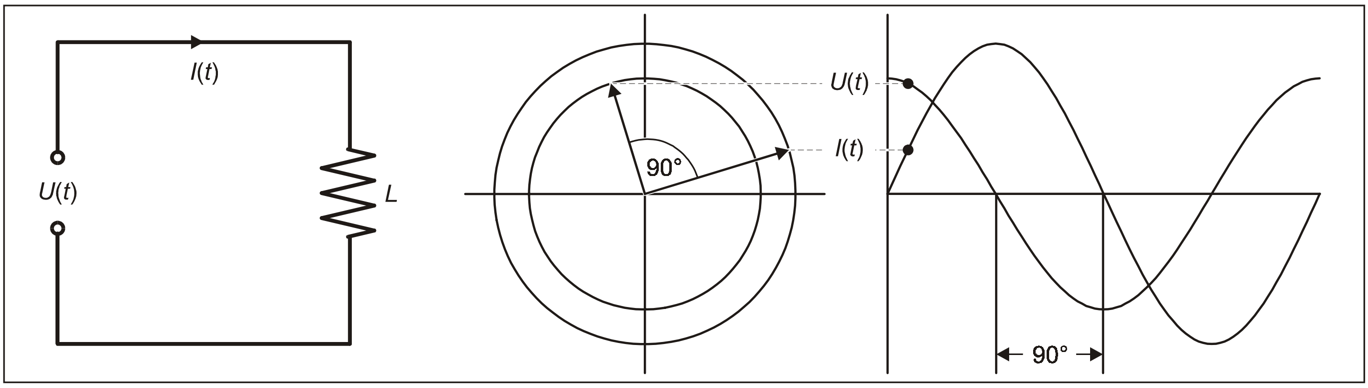
$$\tan \varphi_S = \frac{X_L}{R} \quad \text{(VIII)}.$$

U_S is equal to the voltage U applied, and therefore

$$U_0 = \sqrt{R^2 + X_L^2} \cdot I_0 \quad \text{(IX)},$$

i.e. the series connection of an ohmic resistor and a coil can be assigned the impedance

Fig. 1 AC circuit with a coil (circuit diagram, vector diagram and $U(t)$,$I(t)$ diagram)



Apparatus

1 plug-in board A4	576 74
1 resistor 1 Ω, 2 W, STE 2/19	577 19
1 resistor 100 Ω, 2 W, STE 2/19	577 32
1 Coil with 500 turns STE 2/50	590 83
1 Coil with 1000 turns STE 2/50	590 84
1 function generator S 12	522 621
1 two-channel oscilloscope 303	575 211
2 screened cables BNC/4 mm	575 24
Connecting leads	

$$Z_S = \sqrt{R^2 + X_L^2} \quad \text{(X).}$$

In this arrangement, the voltage is phase-shifted with respect to the current by the angle φ_S (see Fig. 2).

Parallel connection

If the coil is connected in parallel to the ohmic resistor, the same voltage is applied to both of them. The voltage has, for example, the shape given in Eq. (I). The current flowing through the ohmic resistor is

$$I_R = \frac{U_0}{R} \cdot \cos(\omega \cdot t) \quad \text{(XI)}$$

whereas the current flowing through the coil is

$$I_L = \frac{U_0}{X_L} \cdot \cos\left(\omega \cdot t - \frac{\pi}{2}\right) \quad \text{(XII).}$$

The sum of the two currents is

$$I_P = \sqrt{\frac{1}{R^2} + \frac{1}{X_L^2}} \cdot U_0 \cdot cos(\omega \cdot t - \varphi_P) \quad \text{(XIII)}$$

with

$$\tan\varphi_P = \frac{R}{X_L} \quad \text{(XIV).}$$

It corresponds to the total current drawn from the voltage source. Hence, the parallel connection of an ohmic resistor and a coil can be assigned an impedance Z_P, for which the relation

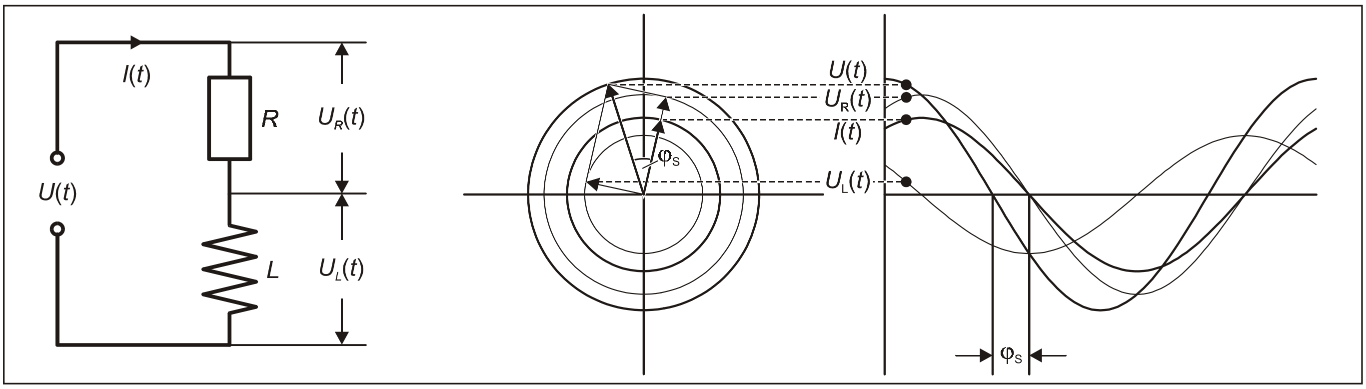
$$\frac{1}{Z_P} = \sqrt{\frac{1}{R^2} + \frac{1}{X_L^2}} \quad \text{(XV)}$$

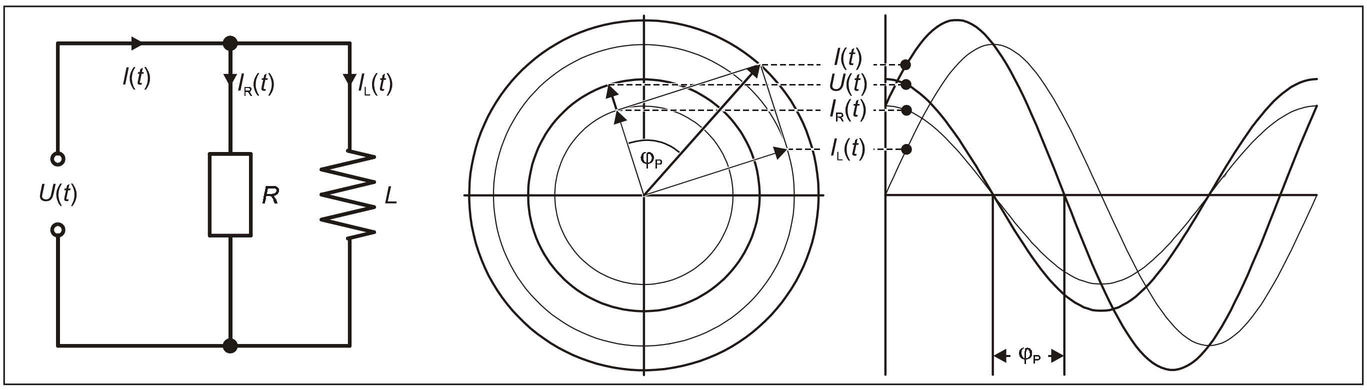
holds. In this arrangement, the voltage is phase-shifted with respect to the current by the angle φ_P (see Fig. 3).

In the experiment, the current $I(t)$ and the voltage $U(t)$ are measured as time-dependent quantities in an AC circuit by means of a two-channel oscilloscope. A function generator is used as a voltage source with variable amplitude U_0 and variable frequency f. From the measured quantities the magnitude of the total impedance Z and the phase shift φ between the voltage and the current are determined.

Fig. 2 AC circuit with a coil and an ohmic resistor in series connection (circuit diagram, vector diagram and $U(t)$, $I(t)$ diagram)

Fig. 3 AC circuit with a coil and an ohmic resistor in parallel connection (circuit diagram, vector diagram and $U(t)$, $I(t)$ diagram)





Setup

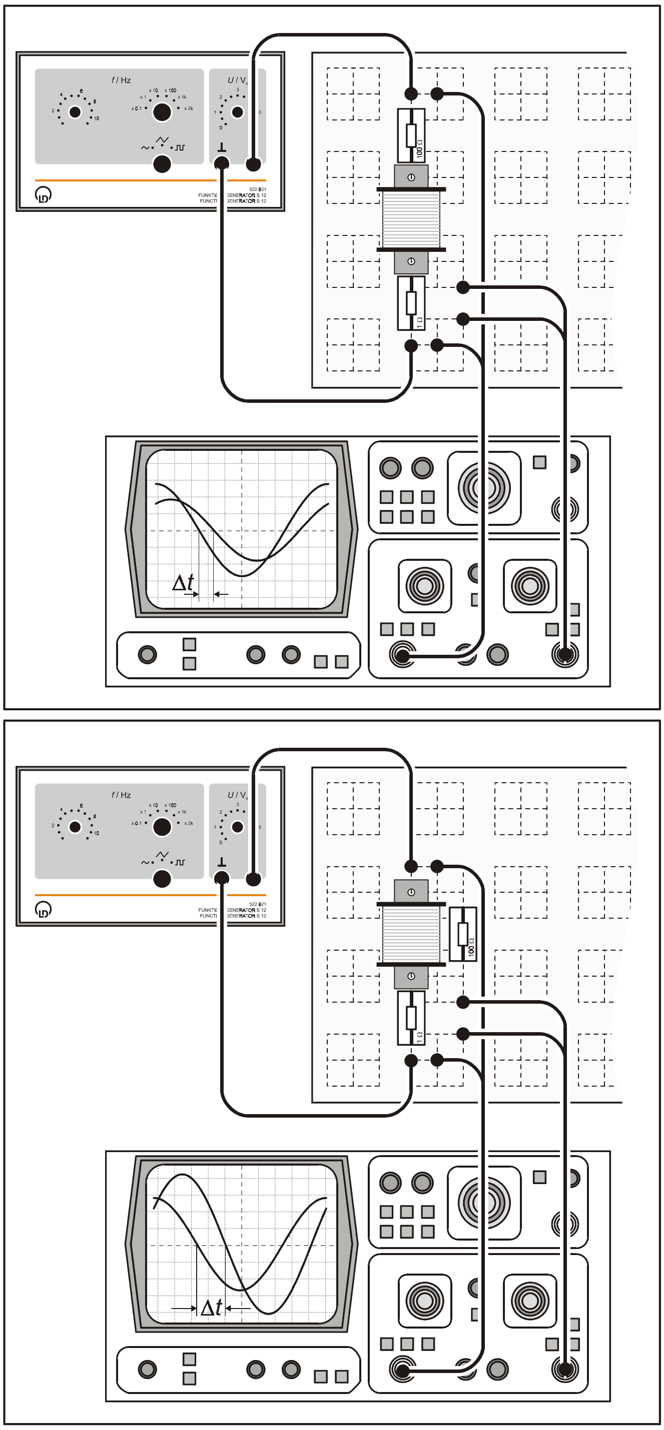
The experimental setup is illustrated in Fig. 4.

– Connect the function generator as an AC voltage source, and select the shape ∿.

– Connect the channel I of the oscilloscope to the output of the function generator, and feed the voltage drop at the measuring resistor 1 Ω into the channel II.

– Press the DUAL pushbutton at the oscilloscope, and select AC for the coupling and the trigger.

Fig. 4 Experimental setup for determining the impedance in circuits with coils and ohmic resistors in series connection (above) and in parallel connection (below)



Carrying out the experiment

– Connect the coil with 1000 turns as an inductance in series with the 100 Ω resistor.

– Switch the function generator on by plugging in the plug-in power supply, and adjust a frequency of 10000 Hz (T = 0.1 ms).

– Select an appropriate time-base sweep at the oscilloscope.

– Adjust an output signal of 5 V.

– Read the amplitude U_m of the signal in the channel II of the oscilloscope, and enter it in the table as current $I_0 = \frac{U_m}{1\,\Omega}$.

– Read the time difference Δt between the zero passages of the two signals.

– Replace the coil by the coil with 500 turns, and repeat the measurement.

– One after another connect the two coils in parallel with the 100 Ω resistor, and repeat the measurement.

– Adjust other frequencies according to Table 1, and repeat the measurements.

Measuring example

U_0 = 5.0 V, R_m = 1 Ω, R = 100 Ω

Tab. 1: measuring data for the frequency f, oscillation period T, number of turns N, time difference Δt and current amplitude I_0

			Series connection		Parallel connection	
$\frac{f}{\text{Hz}}$	$\frac{T}{\text{ms}}$	N	$\frac{I_0}{\text{mA}}$	$\frac{\Delta t}{\text{ms}}$	$\frac{I_0}{\text{mA}}$	$\frac{\Delta t}{\text{ms}}$
10000	0.1	1000	5	0.024	50	0.001
		500	18.5	0.019	54	0.006
5000	0.2	1000	9	0.04	50	0.006
		500	30	0.028	64	0.020
2000	0.5	1000	20	0.08	56	0.032
		500	40	0.04	105	0.08
1000	1	1000	30	0.12	70	0.10
		500	46	0.05	190	0.19
500	2	1000	41	0.16	110	0.28
		500	48	0.04	380	0.36
200	5	1000	42	0.20	210	0.55
		500	46	0.05	520	0.65
100	10	1000	40	0.20	260	0.80
50	20	1000	42	0.0	300	0.5

Evaluation

The measuring data of Table 1 are evaluated as follows:

The phase shift φ is calculated from the time difference Δt between the voltage and the current and from the oscillation period T according to

$$\varphi = 360 \cdot \frac{\Delta t}{T}$$

and the magnitude of the total impedance is obtained from the amplitudes U_0 and I_0 according to

$$Z = \frac{U_0}{I_0}.$$

The results are listed in Table 2, where the inductive reactance of the respective coil calculated according to Eq. (III) is also given. The calculations are based on the following values: L = 4.25 mH for N = 500 and L = 17 mH for N = 1000.

Tab. 2: values of the total impedance Z and the phase shift φ between the voltage and the current calculated from the measuring data from Table 1

			Series connection		Parallel connection	
f / Hz	N	X_L / Ω	Z / Ω	φ	Z / Ω	φ
10000	1000	1068	1000	86°	100	4°
	500	267	270	68°	93	22°
5000	1000	534	556	72°	100	11°
	500	133.5	167	50°	78	36°
2000	1000	214	250	58°	89	23°
	500	53.5	125	29°	48	58°
1000	1000	106.8	167	43°	71	36°
	500	26.7	109	18°	26	68°
500	1000	53.4	122	29°	45	50°
	500	13.35	104	7°	13.2	65°
200	1000	21.4	119	14°	24	40°
	500	5.35	109	4°	9.6	47°
100	1000	10.68	125	7°	19.2	29°
50	1000	5.34	119	0°	16.7	9°

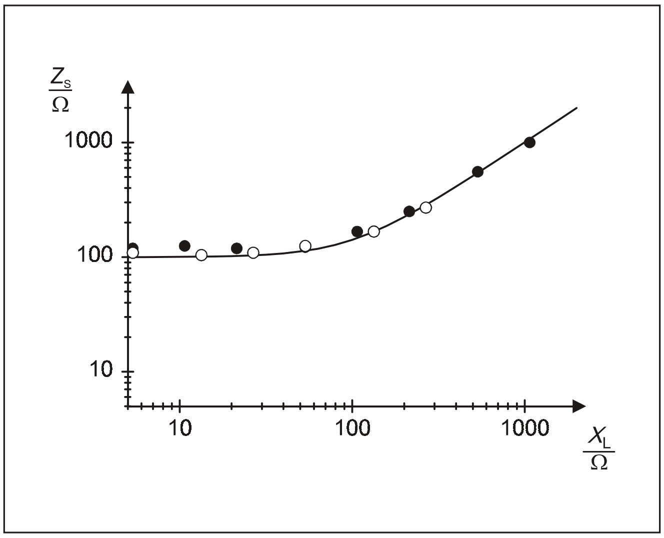


Fig. 5 Total impedance Z_S of the series connection of a coil and a 100 Ω resistor as a function of the inductive reactance X_L
● Measured values for the coil with 1000 turns
○ Measured values for the coil with 500 turns

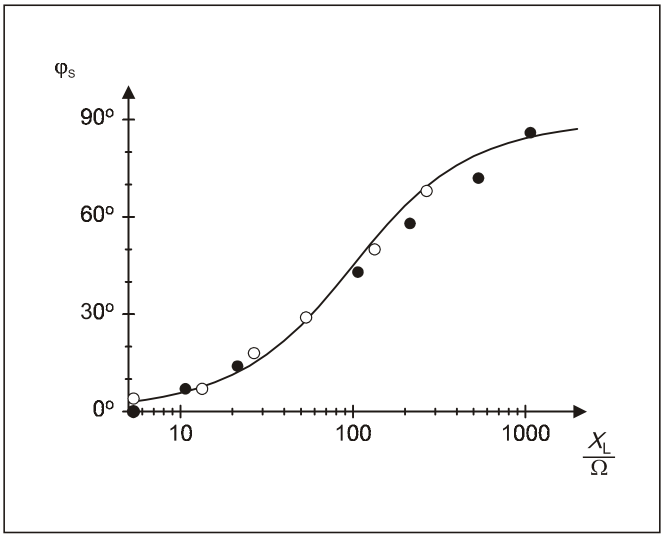


Fig. 6 Phase shift φ_S between the voltage and the current for the series connection of a coil and a 100 Ω resistor as a function of the inductive reactance X_L
● Measured values for the coil with 1000 turns
○ Measured values for the coil with 500 turns

For the series connection, Fig. 5 shows a plot of the impedance Z_S and Fig. 6 shows the phase shift φ_S between the voltage and the current, both as functions of the inductive reactance X_L. The solid lines were calculated according to Eq. (X) and Eq. (VIII), respectively.

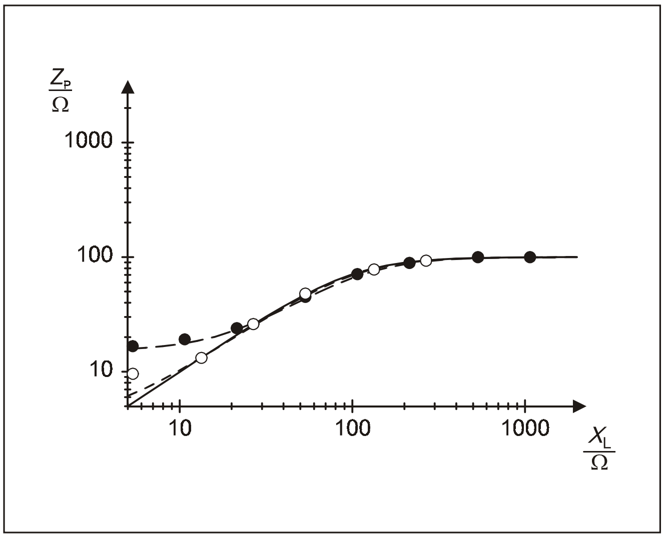


Fig. 7 Total impedance Z_S of the parallel connection of a coil and a 100 Ω resistor as a function of the inductive reactance X_L

● Measured values for the coil with 1000 turns
○ Measured values for the coil with 500 turns
——— calculated for ideal coils
– – – calculated for a real coil with R_L = 18 Ω
- - - - - calculated for a real coil with R_L = 4 Ω

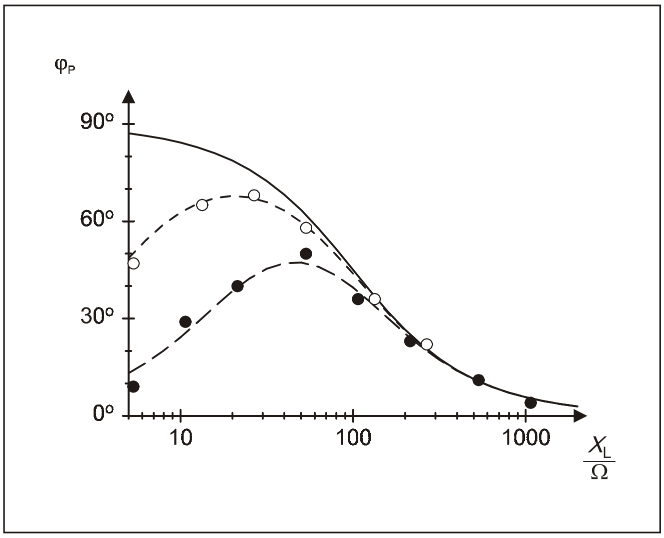


Fig. 8 Phase shift φ_S between the voltage and the current for the parallel connection of a coil and a 100 Ω resistor as a function of the inductive reactance X_L

● Measured values for the coil with 1000 turns
○ Measured values for the coil with 500 turns
——— calculated for ideal coils
– – – calculated for a real coil with R_L = 18 Ω
- - - - - calculated for a real coil with R_L = 4 Ω

The corresponding diagrams for the parallel connection are shown in Figs. 7 and 8. In this case, the solid curves are obtained according to Eqs. (XIV) and (XV). However, they clearly deviate from the measuring results as the ohmic resistance of the two coils was neglected. If an ohmic resistance of R_L = 4 Ω is assumed for the coil with 500 turns and a resistance of R_L = 18 Ω for the coil with 1000 turns, the dashed curves are obtained.

Supplementary information

The mathematical description of the series and parallel connection of an ohmic resistance and an inductive reactance becomes more elegant if complex quantities are considered:

When a voltage

$$U = U_0 \cdot e^{i \cdot \omega \cdot t}$$

is applied to a coil, the inductive reactance is

$$X_L = i \cdot \omega \cdot L .$$

The impedance Z_S of a series connection of an ohmic resistance R and an inductive reactance then is

$$Z_S = R + i \cdot \omega \cdot L .$$

In the case of a parallel connection, the following relation holds for the total impedance Z_P:

$$\frac{1}{Z_P} = \frac{1}{R} + \frac{1}{i \cdot \omega \cdot L} .$$