Confirming Coulomb's law - Measuring with the torsion balance, Schürholz design

Objects of the experiment

- Determination of the force as function of the distance between the charged spheres.
- Determination of the force as function of the amount of charge on the spheres.

Principles

A free standing sphere charged by Q_1 produces a radially symmetric electric field

$$\vec{E} = \frac{1}{4\pi\varepsilon_0} \cdot \frac{Q_1}{r^2} \cdot \frac{\vec{r}}{r} \qquad \text{(I)}$$

$\vec{E}$: electric field of charge Q_1

Q_1: charge of sphere 1

$\varepsilon_0 = 8.85 \cdot 10^{-12}$ As/Vm (permittivity)

r: distance from the center of the sphere

$\frac{\vec{r}}{r}$: unit vector in radial direction from Q_1

A second sphere charged by Q_2 experiences a force $\vec{F}$ when placed in the electric field $\vec{E}$ of charge Q_1:

$$\vec{F} = Q_2 \cdot \vec{E} \qquad \text{(II)}$$

$\vec{E}$: electric field of charge Q_1

Q_2: charge of sphere 2

From equation (I) and (II) we obtain the Coulomb's law

$$\vec{F} = \frac{1}{4\pi\varepsilon_0} \cdot \frac{Q_1 \cdot Q_2}{r^2} \cdot \frac{\vec{r}}{r} \qquad \text{(III)}$$

According equation (III) the force acts in the direction of the connecting lines between the two point charges Q_1 and Q_2.

The sign of the charges Q_1 and Q_2 indicates whether the force F is attractive or repulsive.

Equation (III) holds exact for point charges. Equation (III) can also be applied for charged spheres if the charge distribution on the sphere is uniform and the distance r between the sphere midpoints is significantly greater than the sphere diameter.

In this experiment the Coulomb force between two charged spheres is measured using the torsion balance (Schürholz design). The force is measured as function of the distance r between the spheres (part 1) and as function of the charges Q_1 and Q_2 to confirm equation (III).

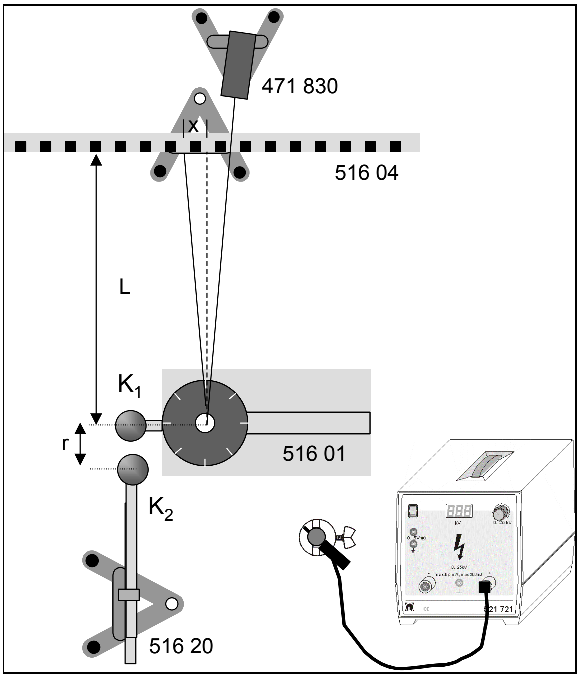

Fig. 1: Schematic diagram of experimental setup (wiring diagram) with the torsion balance to confirm Coulomb's law of electrostatics.

LD Didactic GmbH · Leyboldstrasse 1 · D-50354 Huerth / Germany · Phone: (02233) 604-0 · Fax: (02233) 604-222 · e-mail: info@ld-didactic.de

Apparatus

1 Torsion balance (Schürholz design)	516 01
1 Accessories for Coulomb's law of electrostatics	516 20
1 Scale on stand	516 04
1 High voltage power supply 25 kV	521 721
1 High voltage cable, 1m	501 05
1 Insulated stand rod, 25 cm	590 13
1 Saddle base	300 11
1 Electrometer amplifier	532 14
1 Plug-in unit 230/12V AC	562 791
1 STE capacitor 1nF	578 25
1 STE capacitor 10nF	578 10
1 Multimeter LD analog 20	531 120
1 Faraday's cup	546 12
1 Clamping plug	590 011
1 Connection rod	532 16
2 Stand base, V-shape 20 cm	300 02
1 He-Ne-Laser	471 830
1 Stand rod, 47 cm	300 42
1 Leybold multiclamp	301 01
1 Stop clock	313 07
1 Wooden ruler	311 03
1 Pair cables 50 cm, red/blue	501 45
1 Connection lead, 25 cm, black	500 414
1 Connection lead, 50 cm, black	500 424
2 Connection lead, 100 cm, black	500 444
1 Connection lead, 200 cm, yellow-green	501 43

Setup

- Calibrate the torsion balance see part a.) "Carrying out the experiment". Refer also to the instruction manual 516 01 of the torsion balance.
- Insert the metal sphere K_1 with holder into the sensitive system of the torsion balance.
- Mount the He-Ne-Laser and the scale on stand. Adjust the mirror and the He-Ne-Laser at a distance L of at least 2 m (Fig. 1).
- Place the rod with the second metal sphere K_2 in the adjustable stand.
- Set the marker on the stand guide rod to 3.1 cm and move the stand towards the torsion balance so that the distance between K_1 and K_2 is 1 mm (Fig. 1.). The center points of the spheres are than 3.1 cm apart (sphere diameter 3.0 cm). Thus the marker setting of the guide rod now always gives the distance between the sphere center points. (This only applies if the spheres are not charged. The error of the following experiment when the spheres are charged is up to 2%.)

Carrying out the experiment

Before measuring the torsion balance has to be calibrated. There are two methods (see also instruction sheet 516 01):

a) Calibration of torsion balance (static method)

- For static method lay the balance to one side. The stand rod of the upper plate serves as a support.
- Insert the small calibration rod (length 11 cm) into the rotary body and adjust it horizontally by turning the wheel at the head of the torsion balance. Mark the position of the pointer.
- Insert the weight (m = 0.5 g) at one of the two groves at the tip of the calibration bar. Return the pointer to the previous mark by turning the wheel at the head of the torsion balance and measure the angle α_1.
- Repeat the experiment with the weight on the opposite grove of the calibration bar to determine the angle α_2.
- Now the restoring torque D can be determined according the following relation (Note: The torque of the calibration bar exerts on both wires, which is the reason of the factor 2):

$$M = m \cdot g \cdot b = D \cdot \frac{\alpha}{2}$$

$$D = \frac{2 \cdot m \cdot g \cdot b}{\alpha} \qquad \text{(IV)}$$

m = 0.5 g (calibration mass)

g = 9.81 m/s^2

b = 50 mm (length of calibration bar)

Calibration of torsion balance (dynamic method)

- For dynamic method insert the large calibration rod (length 24 cm) into the rotary body.
- Determine the period of oscillation T (without damping vane) by measuring several times with the stop clock.
- The restoring torque follows from the known oscillation equation:

$$D = 4 \cdot \pi^2 \frac{J}{T^2} \qquad \text{(V)}$$

$$J = \frac{1}{12} \cdot m \cdot \ell^2 = 2{,}72 \cdot 10^{-4}\,\text{kgm}^2 \qquad \text{(moment of inertia)}$$

T: period of oscillation

b) Measuring of the force as function of the distance

- Before measuring the torsion balance has to be calibrated.
- Place the sphere K_2 3.1 mm away from sphere K_1 (center points).
- Now charge both spheres by carefully stroking them with a rubbed plastic rod or transferring the charge from the high voltage power supply. The deflection of the light pointer should be of about 20 cm for a sphere distance r of a about 4 mm.
- Measure the displacement x as function of the various distances r between the charged spheres.
- Measure the distance L between the scale and the mirror.

Note

The above experiment requires sufficient insulation. If the insulation is exceptionally bad, the following may help:

Clean the insulators with a dry cloth and warm air.

In case of very dirty insulators clean with alcohol and then with distilled water and then dry.

Remove any surface charges from the insulator by quickly moving the insulator in and out of a bluish (not yellow, sooting) flame of a Bunsen burner.

c) Measuring of the force as function of the amount of charge

Generally, it is sufficient to demonstrate that the deflection is reduced by half when halving the charge of sphere K_2. Halving the charge is carried out by touching of K_2 by a third equally large uncharged sphere K_3 (not shown in Fig. 1).

- Set the distance r to about 10 cm. The distance should not be too small and the deflection of the light pointer should not be too large.
- Ensure that the isolating rod of K_3 carries no charge; if required discharge the sphere and the rod using a flame (see note above).
- Charge both spheres equally and determine the deflection x of the light pointer the scale.
- Halve the charge of sphere K_2 by touching it with sphere K_3.
- Measure the charge of K_2 and K_3 using the electrometer amplifier (For measuring charges with the electrometer amplifier see instruction sheet 532 14)

Measuring example

a) Calibration of torsion balance (static method)

α_1 = 90°

α_2 = 92°

With $\alpha = \frac{1}{2}(\alpha_1 + \alpha_2) = 1.59\ \text{rad}$

equation (IV) gives: D = 3.09 10^{-4} Nm/rad

Calibration of torsion balance (dynamic method)

Measurements of the period of oscillation T gives an average of

T = 5.83 s.

Using equation (V) gives

D = 3.18 10^{-4} Nm/rad

The angle α is given by (The factor ½ is given by the angle doubling which occurs due to reflection.)

$$\alpha = \frac{x}{2L} \quad \text{(VI)}$$

x: deflection of the light pointer on the scale

L: distance between mirror and scale

From the equation

$$M = F \cdot b = D \cdot \alpha = D\frac{x}{2L}$$

follows the calibration factor:

$$\frac{F}{x} = \frac{D}{2 \cdot L \cdot b} = 15.5 \cdot 10^{-4}\ \frac{N}{m} \quad \text{(VII)}$$

F: force

b: length from center of rotation

x: deflection of the light pointer on the scale

L: distance between mirror and scale

b) Measuring of the force as function of the distance

Table 1: Distance r and deflection x

$\frac{r}{cm}$	$\frac{x}{cm}$
8	40.9
10	24.0
12	16.5
14	12.0
16	9.0
18	7.0
20	5.9

Distance L = 2.05 m

c) Measuring of the force as function of the amount of charge

Table 2: Deflection x for different charges Q_1 and Q_2 at distance of r = 10 cm.

Q_1	Q_2	$\frac{x}{cm}$
Q	Q	26.0
Q	½Q	13.0

Q_2 = 14 nAs

Q_3 = 15 nAs

Evaluation and results

Using equation (VII) allows to determine the electrostatic force F between the charged spheres:

Table 3: Distance r and deflection x

$\frac{r}{cm}$	$\frac{x}{cm}$	$\frac{F * 10^{-5}}{N}$
8	40.9	62.8
10	24.0	37.2
12	16.5	25.6
14	12.0	18.6
16	9.0	14.0
18	7.0	10.9
20	5.9	9.2

Fig. 2 summarizes the result of table 2. The plot shows the dependence of the force F from $1/r^2$. The result of experiment b) and c) together leads to:

1. The electrostatic force is proportional to $1/r^2$
2. The electrostatic force is proportional to product of the charges Q_1 and Q_2.

Thus we can say:

$$F \sim \frac{Q_1 \cdot Q_2}{r^2}$$

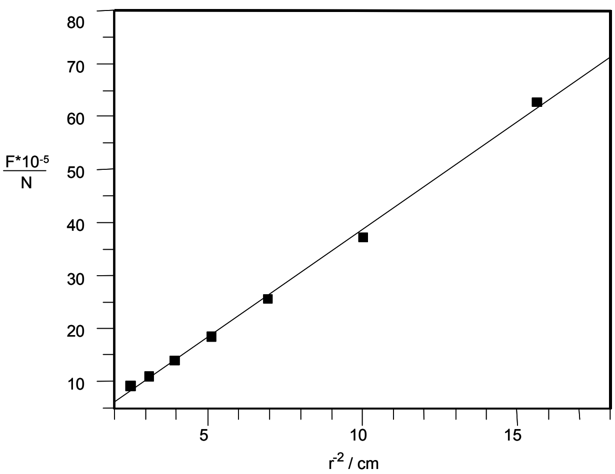

Fig. 2: Electrostatic force as function of $1/r^2$.

Supplementary information

Coulomb's law only applies exactly to point charges. In the case of spheres, mutual interferences prevents uniform distribution of the charge on the sphere. However, if the distance of the center points is sufficiently large this interference may be disregarded.

If in experiment a) the charges Q_1 and Q_2 are chosen to be equal and the charge Q_2 is measured the permittivity ε_0 can be estimated using equation (III):

$$\varepsilon_0 = \frac{1}{4 \cdot \pi} \cdot \frac{Q_1 \cdot Q_2}{r^2} \cdot \frac{1}{F}$$

Table 4: Distance r and deflection x

$\frac{r}{cm}$	$\frac{F * 10^{-5}}{N}$	$\frac{\varepsilon_0 * 10^{-12}}{As / Vm}$
8	62.8	7.92
10	37.2	8.55
12	25.6	8.64
14	18.6	8.72
16	14.0	8.91
18	10.9	9.05
20	9.2	8.69
	Average:	8.64

$$\varepsilon_0 = 8.64 \cdot 10^{-12} \frac{A\,s}{V\,m}$$