Large Projection Apparatus

N. B.

Please observe the recommendations of the label containing safety notes.
Do not allow a finger to enter the danger region while setting or releasing the projection apparatus.
Be careful not to crush any part of your hand.

The large projection apparatus (fig. 1) is intended for quantitative demonstration experiments on the laws of motion of projectiles. The design of the apparatus permits trajectories with initial angles of elevation continuously adjustable from 0° to 90° and with 3 different initial velocities to be produced.

Distances of throw, times of flight and maximum altitudes can be measured in a vivid and clear manner. Since the point at which the projectile is released lies in the axis about which the guide barrel is tilted, the measured distances of throw can be directly compared with one another. In addition, the superposition of two independent motions, namely that of a freely falling ball and that of an obliquely projected ball, can be demonstrated. The use of a light barrier enables the initial velocity of the projected ball to be measured.

Experiments on the conservation of energy can be carried out. For this purpose the plunger can be taken out of the apparatus so that its mass may be determined.

1 Description (see fig. 2)

a) Fixing plate, with

b) 2 holes, enabling the apparatus to be firmly mounted on a base plate by means of countersunk screws.

c) Angle scales, on the front and back, calibrated from 0° (horizontal throw) to 90° (vertical throw).

d) Apertures in scales, enabling the projection apparatus to be clamped down by means of a bench clamp.

e) Apertures in scales for the passage of light when measuring projection velocities by means of a light barrier.

f) Pointer (one on each side).

g) Guide barrel with muzzle-piece connected to the

h) Threaded rod for adjusting the angle of elevation of the guide barrel, with

i) 2 knurled nuts.

j) Spring-loaded plunger with 3 grooves and handle, provided at its front end with a small magnet in order to hold the ball to be projected and at its rear end with

k) Releasing lever for setting the required initial velocity by means of a compression spring located within the guide barrel, and for triggering the throw.

l) Hole to take the hook of a spring balance (100 N) for measuring the force with which the spring has been compressed when using compression-steps 1 and 2.

m) Striking plate for operating the microswitch.

n) Microswitch

o) Terminal sockets for connection to the large electronic stop-clock or to an electromagnet.

p) Toggle-screws for removing the plunger from the guide barrel.

q) Label carrying safety notes.

The standard equipment of the apparatus includes 10 steel balls of 10 mm dia. and 2 steel balls of 30 mm dia.
2 Technical data

Angle of elevation: 0° to 90°, continuously adjustable.

3 different initial velocities \( v_0 \) can be selected:

- using compression-step I: approx. 0.5 m
- using compression-step II: approx. 1 m
- using compression-step III: approx. 1.8 m

Mass of plunger: 285 g ± 5%

Height of point of projectile’s release above the fixing plate: 10 cm

Maximum distances of throw (angle of elevation 45°):

- using compression-step I: approx. 0.5 m
- using compression-step II: approx. 1 m
- using compression-step III: approx. 1.8 m

Scatter: approx. ± 1 to 3 cm longitudinally

approx. ± 1 to 2 cm laterally

The scatter is greater when using compression step III than when using compression-steps I and II.

3 Setting up and using the projection apparatus

3.1 Assembly of the apparatus for oblique and for vertical throw:

The projection apparatus should be set up as shown in fig. 3a at the corner of a table, so that the edge \( (d_1) \) of the rectangular aperture \( (d) \) lies directly over the edge of the table. This ensures that the edge of the table does not interfere with the adjustment of the threaded rod \( (h) \) for vertical throw.

The fixing plate of the projection apparatus should then be secured to the table top by means of bench clamps at the free end of the fixing plate and at the aperture \( (d) \).

The projection apparatus can, however, also be mounted permanently on a base, e.g. a wooden board, by means of countersunk wood screws or bolts with nuts passing through the holes \( (b) \) in the fixing plate \( (a) \).

3.2 Adjustment for horizontal throw; superposition in the case of horizontal throw:

The projection apparatus is set up as shown in fig. 3b. The edge of the table does not interfere, since the angle is not varied but simply adjusted once for all (0°).

3.3 Adjustment of the angle of elevation:

The knurled nuts \( (i) \) (see fig. 2) are loosened and displaced until the guide barrel can be swung to the desired angle. Then both the knurled nuts are tightened firmly enough to ensure that the angle cannot alter, even after several throws have been carried out.

After every series of experiments, and at the latest after each fifth throw, the knurled nuts should be tested for tight fitting and, if necessary, tightened up again.

3.4 Adjustment of the initial velocity of the ball (see fig. 4):

The releasing lever \( (k) \) is first moved to the right. Then the plunger is pulled out of the guide barrel in the direction of the arrow 1 and — depending upon the desired spring compression — the releasing lever is placed in the first, second or third groove of the plunger. In order to keep errors of measurement to a minimum, the plunger should not be rotated during any one series of experiments.

3.5 Carrying out a throw (see fig. 4):

The small ball is placed in the muzzle-piece of the guide barrel so that it lies against the plunger. Then the spring of the projection apparatus is compressed and the releasing lever is operated by hitting it sharply from below with a finger (arrow direction 2). The releasing lever must on no account be pulled slowly out of the groove.

3.6 Removing the plunger in order to determine its mass (see fig. 5):

Unscrew the toggle screws \( (p) \).

Pull part \( (g_1) \) of the guide barrel and the plunger \( (j) \) off the rest of the guide barrel \( (g) \).

Remove the circlip \( (s) \) from the groove \( (u) \) of the plunger \( (j) \) by means of a screwdriver (see detail drawing).

Pull the spring \( (t) \) off the plunger.

Pull part \( (g_1) \) off the plunger.

Determine the mass of the plunger.

Reassembly is carried out in reverse order. While reinserting the circlip \( (s) \) into the groove \( (u) \), the spring \( (t) \) must be slightly compressed.

4 Experiments

General Notes

In order to reduce deviations in the measured values as much as possible, the following points should be noted:

The projection apparatus must always be mounted firmly. The clamping or securing screws should, if necessary, be tightened. The knurled nuts for adjusting the angle of elevation, too, should be retightened after a few throws.
When pulling out the plunger to compress the spring, care should be taken to ensure that the plunger retains the same position throughout any one series of measurements, i.e. that it is not rotated about its axis between successive throws. The measured values are dependent on the plunger's position.

The releasing lever must never be lifted slowly out of the groove in the plunger; it should always be struck sharply and firmly from below.

4.1 Determination of the range

The determination of the ranges $s$ at different angles $\alpha$ and initial velocities $v_0$ is carried out as shown in fig. 6a.

**Apparatus:**

- 2 bench clamps .................................. 301 06
- Strip of carbon paper, 1.5 m long, 10 cm wide
- Transparent graph-paper, 1.5 m long, 10 cm wide or tracing paper, 1.5 m long, 10 cm wide.
- Adhesive tape for sticking the various strips together.
- If tracing paper is used, a steel tape measure (311 77) must also be available.
- 1 supporting platform, e.g. of wood, 10 cm high; about 1.2 m long and 12 cm wide.

**Note:**

The advantage of using 1.5 m long strips is that the various ranges are recorded on a single sheet of paper and can thus be directly compared with each other. It is, of course, perfectly possible to use separate sheets, measuring the ranges separately with the tape measure.

**Carrying out the experiment:**

The supporting platform is set up in front of the projection apparatus in the direction of throw, and the strip of carbon paper is placed on it, or attached to it with adhesive tape, with its pigmented side upwards.

The transparent graph-paper or thin tracing paper is then placed over the carbon paper and secured with adhesive tape.

Now the projection apparatus is adjusted and operated, and the ranges corresponding to the various angles of elevation and initial velocities can be measured on the strip of graph-paper or tracing paper. The balls mark the points where they strike the platform unambiguously by producing small black points. It is a good idea to number the points of arrival in the sequence of the throws. In this way, systematic deviations (e.g. due to a loosening of the knurled nuts) can be rapidly detected.

The diagram (fig. 6b) displays the functional relationships between the angle of elevation $\alpha$, the range $s$ and the initial velocity $v_0$ for the three possible stages of compression I, II and III. The parameter for this set of curves is a function of the initial velocity.

![Graph showing ranges as a function of angles of elevation for the three compression-steps I, II and III.](image)

4.2 Measurement of the time of flight

The experiment is set up as shown in fig. 7a in order to allow the time of flight $t$ to be measured for various values of the angle of elevation $\alpha$ and initial velocity $v_0$. 

![Diagram showing experimental setup](image)
III are used, the large contact plate must be displaced in the direction of throw.

The functional relationship between time of flight, angle of elevation and initial velocity are displayed in the diagram fig. 7b. Also in this case the parameter for the set of curves is a function of the initial velocity.

4.3 Observing projectile motions using a stroboscope
To make projectile motions visible, a stroboscope is used. Assembly is according to Fig. 8.

4.4 Plotting the parabolic trajectory
The arrangement for plotting the trajectory of the ball under various conditions is shown in fig. 9.

As soon as the ball strikes the contact plate, the digital counter is stopped enabling the time of flight to be read off.

If other values of the angle of elevation $\alpha$ and other initial velocities $v_0$ corresponding to the compression steps I, II and
Carrying out the experiment:

The carbon paper is attached, pigmented side uppermost, to the reflection plate by means of adhesive tape. The transparent graph paper or thin tracing paper is stuck on over the carbon paper.

The plate with stand rod, e.g. the reflection plate, is clamped into the saddle base and set up in front of the projection apparatus with its left-hand edge in the line of throw.

The two tape measures or wooden rulers are placed along the edges of the saddle base, as shown in the illustration.

In order to plot a trajectory, the plate is moved stepwise (e.g. in steps of 2 cm or 5 cm) between every two throws, both longitudinally (arrow direction 1) and laterally, at right angles to the former (arrow direction 2).

In this way, marks corresponding to the points at which the ball has struck the plate are obtained on the graph-paper or tracing paper, and a parabolic curve corresponding to the trajectory can be drawn through these marks (see graph contained in fig. 9).

In place of the two tape measures or rulers, a large sheet of graph paper or squared paper can be attached to the table top in order to determine the displacements of the reflection plate in its saddle base.

4.5 Demonstration of the superposition of motions

4.5.1 Superposition of motions in the case of oblique throw

If a projectile is thrown vertically or obliquely, the vertical component of its velocity at any instant is formed by the vectorial addition of the vertical component of its initial velocity and the vertically downward velocity produced by free fall under gravity.

The mutually non-interfering superposition of the two motions in the case of oblique throw can be demonstrated as shown in fig. 10.
Carrying out the experiment:
The holding magnet is connected to the low-tension tapping transformer via the microswitch on the projection apparatus. The contact on the projection apparatus must initially be closed, i.e. the plunger of the projection apparatus must be pulled out against the force of the spring.

The large ball is then suspended from the electromagnet, i.e. 0.5 m from the point of release of the small ball in the projection apparatus, and in the straight line formed by producing the axis of the guide barrel of the projection apparatus outwards.

The location of the point of suspension of the large ball should be determined by means of a long ruler, using the two flat surfaces on the guide barrel (see detail drawing in fig. 10).

The electromagnet’s circuit is closed via the switch of the projection apparatus when the plunger of the projection apparatus is pulled out. At the instant of release of the small ball by the projection apparatus, the circuit of the magnet holding the large ball is automatically broken by the micro-switch, so that the freely falling ball and the obliquely thrown one start their motions simultaneously. If the projection apparatus has been carefully adjusted, the freely falling ball will be struck by the thrown ball at the point of intersection of the two paths. The collision can be more clearly observed if the balls are illuminated and observed by reflected light, or if the stroboscope (see experiment 4.3) is used for observation of the motions.

4.5.2 Superposition of motions in the case of horizontal throw, demonstration of a thrown and a falling ball striking a horizontal surface simultaneously

The demonstration of the superposition of motions in the case of horizontal throw, i.e. the demonstration that a ball thrown horizontally and a ball allowed to fall freely from the same height will strike a horizontal surface simultaneously, is carried out by means of the arrangement shown in fig. 11.

**Apparatus**

<table>
<thead>
<tr>
<th>Item</th>
<th>Cat. No.</th>
</tr>
</thead>
<tbody>
<tr>
<td>3 Bench clamps</td>
<td>301 06</td>
</tr>
<tr>
<td>1 Holding magnet</td>
<td>336 21</td>
</tr>
<tr>
<td>1 Voltage source, 6 V d.c., 0.5 A, e.g.</td>
<td>591 09</td>
</tr>
<tr>
<td>2 Stand rods, 25 cm</td>
<td>300 41</td>
</tr>
<tr>
<td>1 LEYBOLD multiclamp</td>
<td>301 01</td>
</tr>
<tr>
<td>3 Connecting leads</td>
<td>from 501 20 to 501 43</td>
</tr>
</tbody>
</table>

Carrying out the experiment:
The projection apparatus is adjusted for horizontal throw (angle of elevation 0°) and the electromagnet with the second small ball is set up in front of it in such a manner that this ball is held at the height of the point of release of the ball in the projection apparatus, i.e. at the height of the ball which is to be thrown horizontally.

The circuit is connected up as in experiment 4.5.1.

If the projection apparatus is now activated, the ball hanging from the electromagnet begins to fall freely under gravity at the same instant as the other ball, contained in the projection apparatus is thrown. The two balls will hit the floor simultaneously. If the balls are allowed to fall onto two metal trays, the simultaneous striking of the two balls can be made audible. The disadvantage of this experiment is that the ear cannot resolve two events separated by a time interval of approximately 0.1 s or less.

4.6 Determination of the mass of the plunger and of the spring constant for the purpose of calculating the initial velocity by means of the law of conservation of energy

In order to be able to determine the value of the initial velocity \( v_0 \) corresponding to the different degrees of spring compression by means of the law of conservation of energy, it is necessary to determine the masses of the plunger, the effective parts of the spring and the ball \( m_p, m_s \) and \( m_b \), respectively, and to obtain the spring constant \( D \) from the force \( F \) compressing the spring and the distance \( s \) by which the spring is compressed.

**Apparatus**

<table>
<thead>
<tr>
<th>Item</th>
<th>Cat. No.</th>
</tr>
</thead>
<tbody>
<tr>
<td>2 Bench clamps</td>
<td>301 06</td>
</tr>
<tr>
<td>2 Dynamometers, 100 N.</td>
<td>314 201</td>
</tr>
<tr>
<td>1 Callipers</td>
<td>311 52 or 311 54</td>
</tr>
</tbody>
</table>

In addition:

<table>
<thead>
<tr>
<th>Item</th>
<th>Cat. No.</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 Screw-driver for dismantling the projection apparatus</td>
<td></td>
</tr>
</tbody>
</table>

Carrying out the experiment:
The force of compression \( F \) in the compression-steps I and II can be measured by means of a 100 N dynamometer hooked into the hole provided for this purpose in the handle of the plunger (see fig. 12). In order to be able to measure the force of compression in compression-step III, two 100 N dynamometers must be used in parallel; these should be hooked onto the two arms of the plunger handle.

The shortening of the spring obtained in each of the three compression-steps is determined from the distance between the striking plate and the guide barrel, using the vernier callipers. The mass of the plunger can be found after dismantling the projection apparatus as shown in fig. 5.
The energy stored in the compressed spring can now be calculated from the formula.

$$W_S = \frac{1}{2} D s^2.$$ 

This energy is transformed into the kinetic energy of the plunger, the ball and parts of the spring, which can be calculated as follows:

$$W_K = \frac{1}{2} (m_p + m_b + m_S) v_o^2.$$ 

Since $$W_K = W_S$$, the initial velocity of the ball can be calculated:

$$v_o = \sqrt{\frac{D s^2}{m_p + m_b + m_S}} = \sqrt{\frac{F_s}{m_p + m_b + m_S}}.$$ 

As an approximation, the value $$m = 0.3 \text{ kg}$$ will be used in the following example for $$m_p + m_b + m_S$$. The values of the initial velocity $$v_o$$ can then be read off from the following table:

<table>
<thead>
<tr>
<th>Compression-step</th>
<th>Force of compression $$F$$ (N)</th>
<th>Distance compressed $$s$$ (m)</th>
<th>$$v_o = \sqrt{\frac{F_s}{m}}$$ (m/s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>I</td>
<td>70</td>
<td>0.02</td>
<td>2.16</td>
</tr>
<tr>
<td>II</td>
<td>105</td>
<td>0.03</td>
<td>3.24</td>
</tr>
<tr>
<td>III</td>
<td>185</td>
<td>0.04</td>
<td>4.55</td>
</tr>
</tbody>
</table>

Only 1/3 of the mass of the spring is effective as the following equation applies

$$\int_0^\ell \frac{1}{2} \mu F \left( \frac{v_o}{x} \right)^2 dx = \frac{1}{2} \mu F \left( \frac{v_o}{\ell} \right)^2 \int_0^\ell x^2 dx = \frac{1}{2} \mu F \frac{\ell^2}{3} v^2.$$ 

$$\mu_F =$$ mass of the effective part of the spring; $$\ell =$$ total length of the spring (without compression).

**4.7 Determination of the initial velocity using a light barrier**

A digital counter is used to measure the time during which the flying ball interrupts a light barrier in each of the compression-steps in turn (fig. 13). From this time and the diameter of the ball (10 mm) the corresponding values of the initial velocity $$v_o$$ of the flying ball are determined.

**Carrying out the experiment:**

Set switch ① of the digital counter on 'start'. Prior to each measurement, press the resetting push button. For setting into operation refer to the Instruction Sheet 575 50.

The initial velocities for horizontal throw (angle of elevation 0°) can be determined for all the three compression-steps.

<table>
<thead>
<tr>
<th>Compression-step</th>
<th>Period of occlusion of the photodiode $$t$$ (s)</th>
<th>Initial velocity of the ball $$v_o$$ (m/s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>I</td>
<td>$$4.7 \times 10^{-3}$$</td>
<td>2.1</td>
</tr>
<tr>
<td>II</td>
<td>$$3.0 \times 10^{-3}$$</td>
<td>3.3</td>
</tr>
<tr>
<td>III</td>
<td>$$2.3 \times 10^{-3}$$</td>
<td>4.3</td>
</tr>
</tbody>
</table>

**4.8 Determination of maximum altitudes of trajectories**

The apparatus is set up as shown in fig. 14 in order to determine the maximum height reached by the ball thrown vertically.

**Apparatus:**

<table>
<thead>
<tr>
<th>Cat. No.</th>
<th>2 Bench clamps</th>
<th>301 06</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 Vertical scale</td>
<td>311 22</td>
<td></td>
</tr>
</tbody>
</table>

In addition:

1 Strip of sheet metal bent through 90° approximately 20 mm from one end, so that it can be attached behind the slider of the vertical scale (see detail drawing in fig. 14).

**Carrying out the experiment:**

The strip of sheet metal inserted behind the slider of the vertical scale is displaced vertically until a ball thrown vertically upwards from the projection apparatus just fails to strike it.
The height reached by the ball can then be read off on the vertical scale. The functional relationships between altitude, angle of elevation and initial velocities are shown in the graph included in fig. 14. The parameter for this set of curves is a function of the initial velocity.

In order to measure the maximum altitude of the trajectory in the case of oblique throw, the vertical scale must also be displaced horizontally in the direction of throw, so that the point of maximum altitude, where the ball’s vertical component of motion changes direction from upwards to downwards, can be located for various angles of elevation and various initial velocities.

The initial velocities of the ball can be calculated from the altitudes reached by the ball when thrown vertically, using the formula

\[ v_o = \sqrt{2gh}. \]

On determining the initial velocity \( v_o \) in the case of vertical throw upwards, a systematic deviation tending towards minor values results contrary to the results of previous experiments. The vertical throw upwards is a uniformly decelerated motion, as the following equation applies.

\[ v = v_o - gt = \sqrt{2gh - gt} \]

**Example:**

<table>
<thead>
<tr>
<th>Compression-step</th>
<th>Altitude ( h ) (cm)</th>
<th>Initial velocity ( v_o ) (m/s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>I</td>
<td>22</td>
<td>2.08</td>
</tr>
<tr>
<td>II</td>
<td>50</td>
<td>3.13</td>
</tr>
<tr>
<td>III</td>
<td>87</td>
<td>4.13</td>
</tr>
</tbody>
</table>