

## Investigating the elastic and plastic extension of metal wires

Recording and evaluating with CASSY

### Objects of the experiment

- Recording the stress-strain diagram of a copper wire and an iron wire
- Determining the elastic modulus from the proportionality region

### Principles

If a wire is strained by a tensile force it expands. If the strain is below the so-called elastic limit EL (Fig. 1) the wire returns to its original length after the strain has been removed.

The expansion in the elastic range is given by the relative change of length:

$$\varepsilon = \frac{\Delta L}{L} \quad (I)$$

$\varepsilon$ : expansion

$\Delta L$ : elongation

$L$ : initial length

The strain  $\sigma$  is given by the force  $F$  which acts perpendicular on the cross sectional area  $A$  of the wire:

$$\sigma = \frac{F}{A} \quad (II)$$

$F$ : normal force

$A$ : cross-sectional area

If the deformations are small in comparison to the dimensions of the wire the expansion  $\varepsilon$  is proportional to the strain  $\sigma$ :

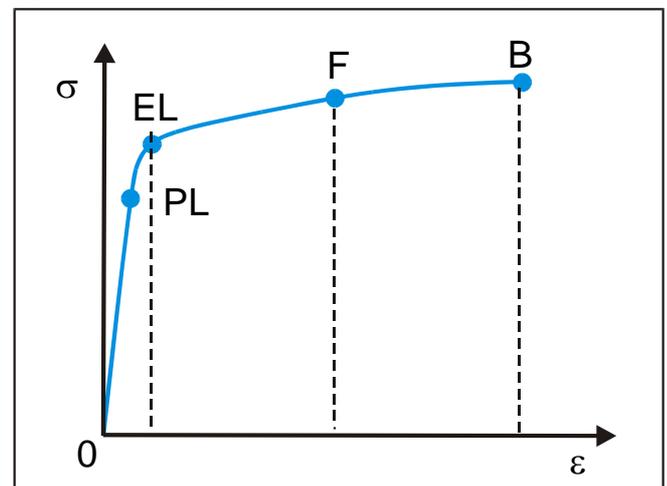
$$\sigma = E \cdot \varepsilon \quad (III)$$

The proportionality constant  $E$  depends on the material and is called the elastic modulus.

For most metallic materials Hooke's law (equation (III)) is valid throughout the elastic range (Fig.1). i.e. where the expansion is linearly proportional to its tensile stress. For some materials, e.g. aluminium, Hooke's law is only valid for a portion of the elastic range. For these materials a proportional limit PL (Fig. 1) is defined, i.e. the range in which the errors associated with the linear approximation are negligible.

If the strain exceeds this proportional limit PL the deformations are not proportional to the strain. However, the deformations still disappear between the proportional limit and the elastic limit after the strain has been relieved. Beyond the elastic limit EL, permanent deformation will occur. The elastic limit corresponds to the lowest stress at which permanent deformation can be measured.

Fig. 1: Schematic representation of stress-strain-diagram of a metal



During further expansion the strain increases. When the flow point F has been reached a small increase in strain causes a large increase in expansion. Then follows a range where the cross-section decreases significantly until the sample breaks at point B.

In this experiment an iron wire and a copper wire are elongated by turning a wheel. The elongation  $\Delta L$  is measured by the Rotary Motion Sensor S. The tensile force F is measured by the Force Sensor S.

### Apparatus

1 Copper wire, 0.20 mm $\varnothing$ .....	550 35
1 Iron wire, 0.20 mm $\varnothing$ .....	550 51
1 Sensor CASSY S .....	524 010USB
1 CASSY Lab .....	524 200
1 Force Sensor S .....	524 042
1 Rotary Motion Sensor S .....	524 082
2 Simple bench clamp .....	301 07
3 Leybold multiclamp .....	301 01
2 Stand rod, 25 cm .....	301 26
1 Stand rod, 100 cm .....	300 44
1 Tape measure, 1 m/1 mm .....	311 78

*additionally required:*

PC with Windows 98/NT or higher

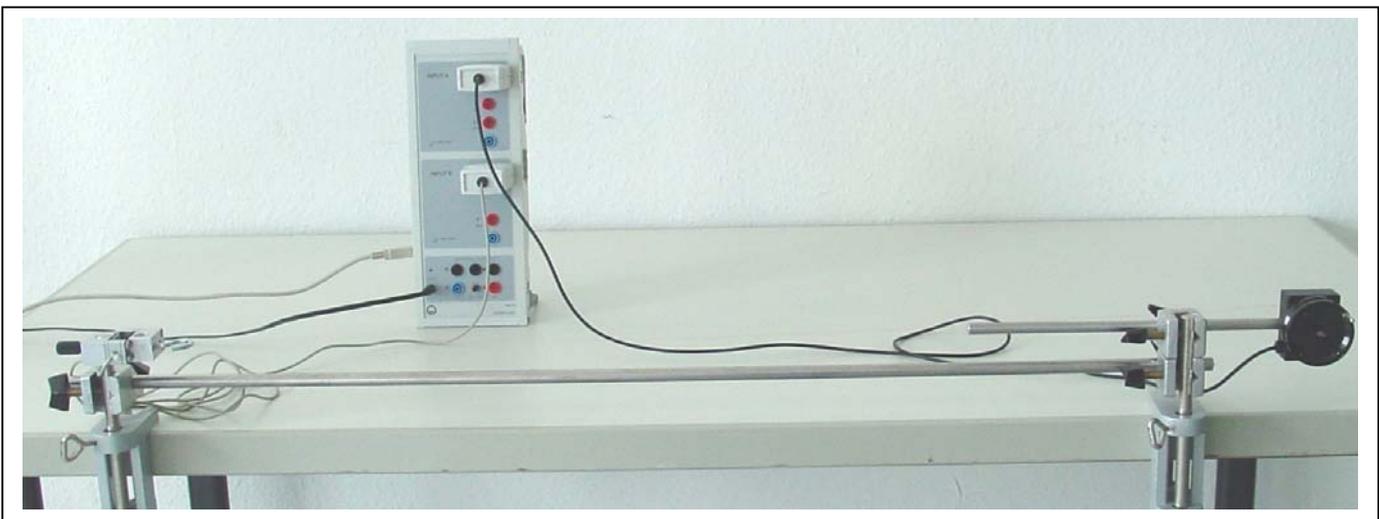
### Setup

#### Setting up the experiment

The experimental setup is shown in Fig. 2.

- Mount the simple bench clamps to the table. The distance between the simple bench clamps should be about 95 cm.
- Fix a stand rod 25 cm to each simple bench clamp in such a manner that two Leybold multiclips can be fixed to it (Fig. 2).
- Fix the stand rod 100 cm with the aid of the Leybold multiclips to each stand rod 25 cm in simple bench clamps.
- Fix the force sensor with the aid of a Leybold multiclip to the left end of the stand rod 100 cm as shown Fig. 3.
- Fix the Rotary Motion Sensor S opposite to the force sensor S as shown in Fig. 4.

Fig. 2: Experimental setup to record the stress-strain-diagram of a copper and a iron wire.

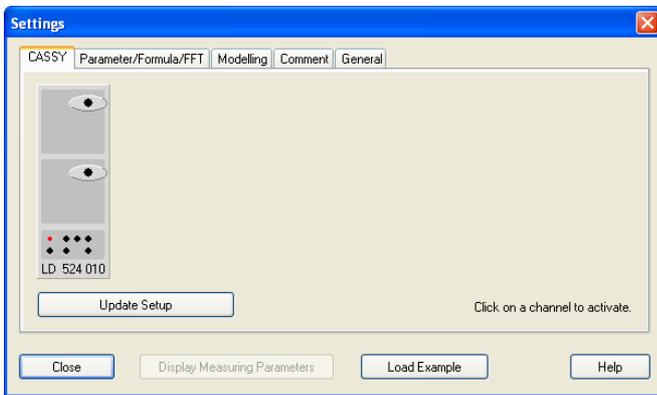


**Setting up the CASSY measuring system**

- Connect the Rotary Motion Sensor S to the “Input A” of Sensor CASSY.
- Connect the Force Sensor S to the “Input B” of Sensor CASSY.
- If not yet installed install the software CASSY Lab and open the software.
- Open the window “Settings” using the tool box button  or function key F5 from the top button bar:

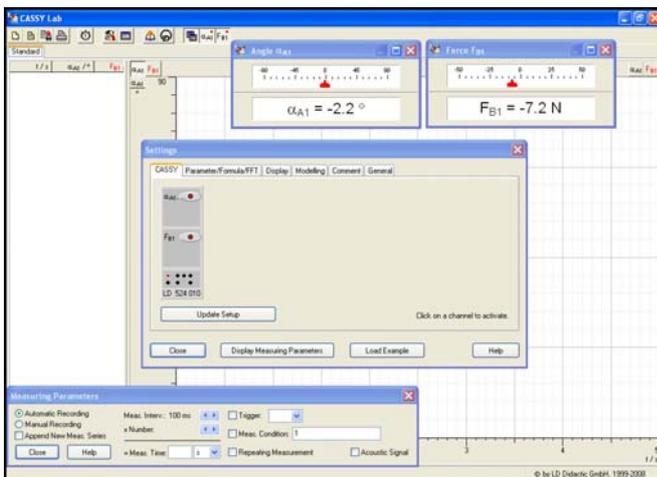


- Sensor CASSY with the connected Sensors, i.e. Rotary Motion Sensor S at “Input A” and Force Sensor S at Input B should be displayed at the tab “CASSY” if Sensor CASSY is connected via the USB port to the computer.

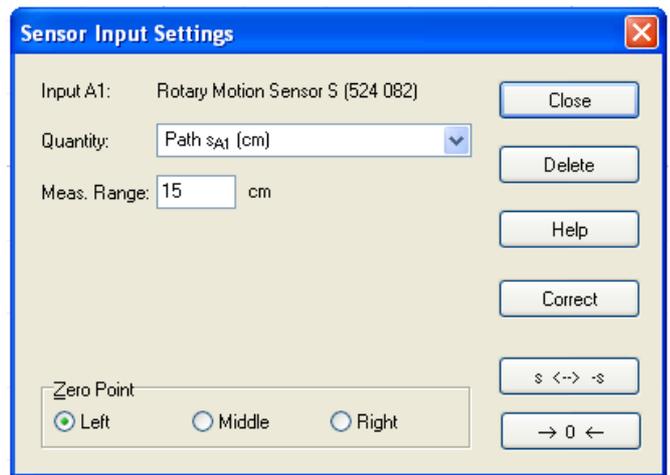


- Activate connected sensors by clicking at the Rotary Motion Sensor S and the Force Sensor S.

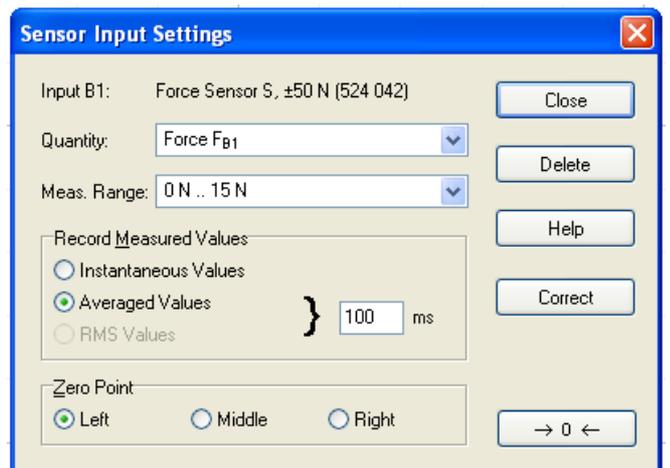
*Note: Further details about connecting sensors to Sensor CASSY can be found in the manual “CASSY Lab Quick Start”.*



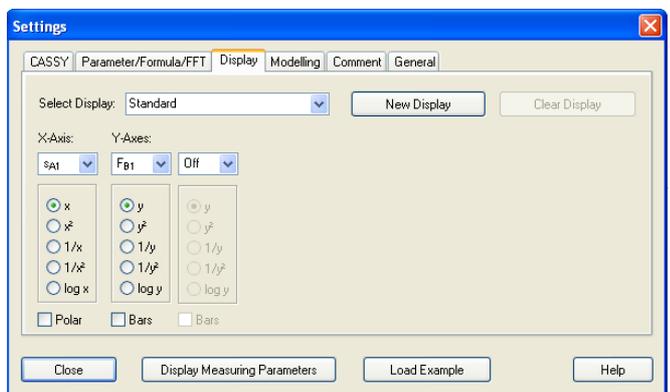
- In the window “Measuring Parameters” set the measuring interval to 500 ms.
- Click with the right mouse button on the window “Angle  $\alpha_{A1}$ ” or the corresponding button in the top menu bar. Select “Path  $s_{A1}$ ” and Zero point “left”.



- Turn the wheel of the Rotary Motion Sensor S to see whether the directions has to be switched by the button “s  $\leftrightarrow$  -s”.
- Click with the right mouse button on the window “Force  $F_{B1}$ ” or the corresponding button in the top menu bar. Select measurement range 0 N to 15 N.



- Select averaged value and Zero point “left”.
- Select the tab “Display” in the window “Settings”
- Then set the X-axis to “ $s_{A1}$ ” and the Y-axis to “ $F_{B1}$ ”



- Close all windows.

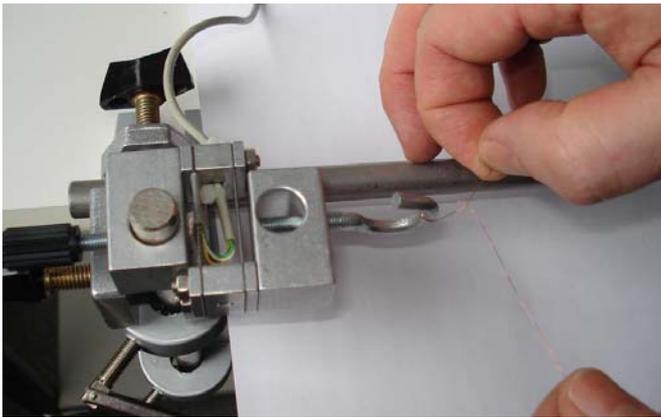


Fig. 3: How to fix a wire to the Force Sensor S

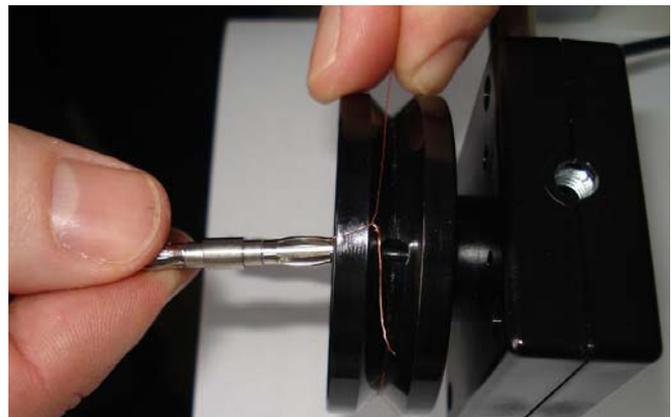
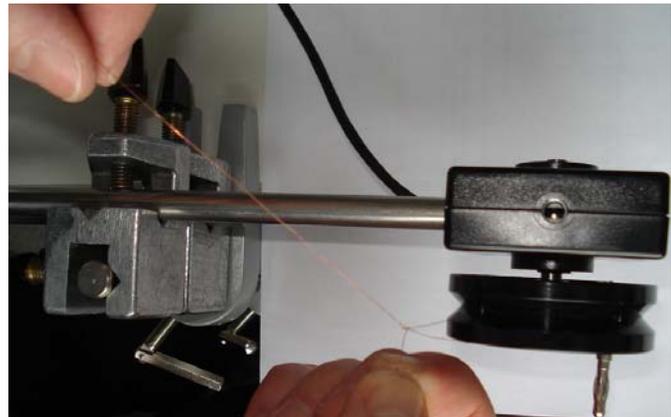


Fig. 4: How to fix a wire to the wheel of the Rotary Motion Sensor S:   
top: attaching the copper wire to a hole of the wheel.   
bottom: fixing the wire with the plug-in axle.

### Carrying out the experiment

- Cut off an approx. 110 cm long piece of iron wire.   
*Important:* The wire must not be bent at any point.
- Fix the wire a to the hook of the Force Sensor S as shown in Fig. 3.
- Fix the other end of the wire to the wheel of the Rotary Motion Sensor S using the 4 mm plug ( Fig. 4).
- Measure the initial length of the wire.
- Click with the right mouse button into the window "Force FB1" to reset the zero position with the button  $\rightarrow 0 \leftarrow$  (repeat as required).
- Click with the right mouse button into the window "Path sA1" to reset the zero position with the button  $\rightarrow 0 \leftarrow$  (repeat as required).

*Note: After you have fixed the wire to the wheel of the Rotary Motion Sensor S the wheel should not be released.*

- Start the measurement with the button  or function key F9 to start recording.
- Slowly turn the wheel to apply strain until the wire breaks. While turning the wheel the appropriate speed of data recording can be controlled by watching the screen.
- When the wire breaks stop data recording. Repeat the measurement for the copper wire.

*Note: The button  works as a toggle switch. The data acquisition can be stopped by pressing  or F9. When selecting "Manual Recording" in the window "Measuring Parameters" point by point can be measured by turning the wheel of the Rotary Motion Sensor S stepwise and pressing the short cut key F9.*



*Note: The measurement can be saved by pressing the button  or using the function key F2.*

### Measuring example

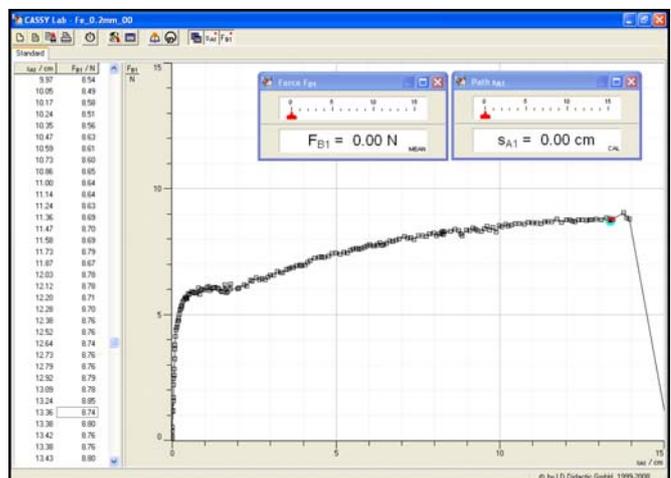


Fig. 5: Force as a function of the elongation of a iron wire.

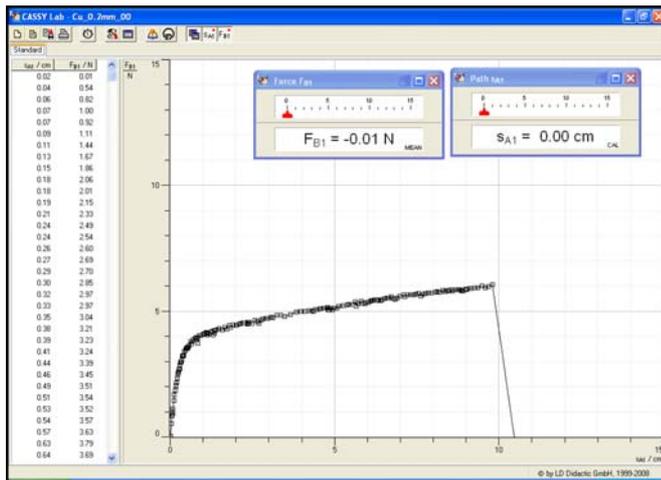


Fig. 6: Force as a function of the elongation of a copper wire.

## Evaluation and results

To determine the elastic modulus  $E$  according to equation (III) the expansion  $\varepsilon$  and the strain  $\sigma$  have to be evaluated and plotted in a diagram. A step-by-step procedure is given to define and plot the required quantities:

### CASSY: Defining the expansion $\varepsilon$

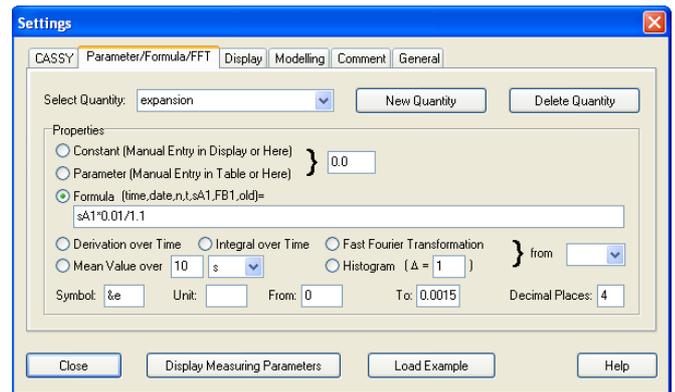
- If the window "Settings" is not open press the button or function key F5 and select the tab "Parameter/Formula/FFT" (Fig. 7)
- Click on the button "New Quantity" to define the expansion  $\varepsilon$  (Fig. 7).
- Select "Formula" and enter (compare equation (I) with a fixed length of  $L = 1.01$  m):

$$sA1 * 0,01 / 1.01$$

Notes:

The symbol  $\varepsilon$  is displayed as "&e". How to enter Greek letters can be found in the appendix of the manual "CASSY Lab Quick Start".

- Enter "&e" to define the symbol.
- As the relative elongation  $\Delta L/L$  has no dimension leave the field "Unit" empty.
- Enter the values for the range, i.e. "0" to "0.15" and choose 4 for decimal places.

Fig. 7: Defining the formula for the expansion  $\varepsilon$  by using the new quantity tool (function key F5).

### CASSY: Defining the strain $\sigma$

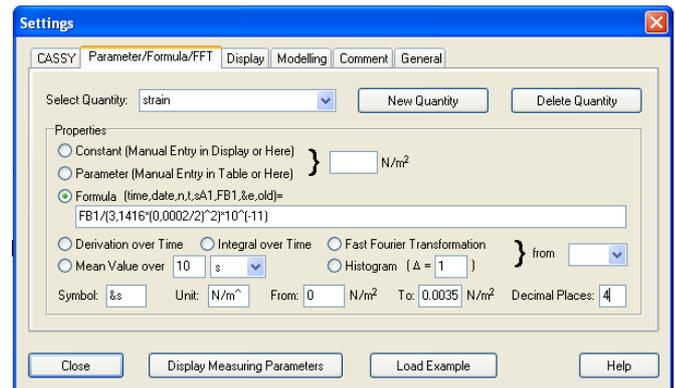
- If the window "Settings" is not open press the button or function key F5 and select the tab "Parameter/Formula/FFT" (Fig. 8)
- Click on the button "New Quantity" to define the strain  $\sigma$ .
- Select "Formula" and enter (compare equation (II) with  $A = \pi \cdot r^2$ ):

$$FB1 / (3,1416 * (0,0002/2)^2) * 10^{(-11)}$$

Notes:

The symbol  $\sigma$  is displayed as "&s". How to enter Greek letters can be found in the appendix of the manual "CASSY Lab Quick Start".

- Enter "&s" to define the symbol.
- Enter "N/m^2" for the unit.
- Enter the values for the range, i.e. "0" to "0.0035" and choose 4 decimal places.

Fig. 8: Defining the formula for the strain  $\sigma$  by using the new quantity tool (function key F5).

**CASSY: Plotting the strain  $\sigma$  as function of the expansion  $\epsilon$**

To plot the strain  $\sigma$  as function of expansion  $\epsilon$  proceed as follows:

- Press the button  or function key F5 and select the tab "Display" (Fig. 9).
- Click on the button "New Display"
- Enter a name, e.g. "stress-strain-diagram"
- Select " $\epsilon$ " for the X-axis and " $\sigma$ " for the Y-axis

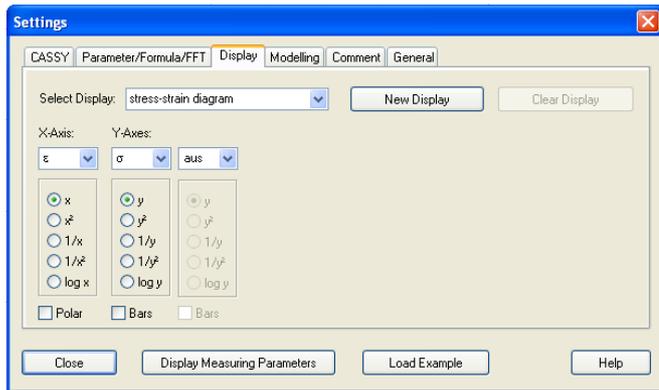


Fig. 9: Defining a new display "stress-strain-diagram" to plot the strain  $\sigma$  as function of expansion  $\epsilon$  by using the new display tool (function key F5).

- Close the window "Settings" and select the tab "stress-strain-diagram".

*Note: Further details about "Changing graphical settings" and "Performing data evaluations" can be found in the manual "CASSY Lab Quick Start".*

**CASSY: Data evaluation**

- Click with the right mouse button into the graph to select the stress-strain diagram.
- Click with the right mouse button into the graph to access the "Fit Function" tool. Chose "Fit Function" and select "Straight Line through Origin".
- Select the data to be fitted. Hold down the left mouse button and drag the mouse pointer over the data points in the proportionality range (selected data become blue) to determine the elastic modulus E in equation (III).

*Note: You may display the evaluation result (shown in the status line at the bottom of the window) in the plot using short cut key Alt-T.*

Example of a measurement of the stress as a function of the expansion of a iron and copper wire are shown in Fig. 10. The fit of a straight line through the origin in the proportional region gives the elastic modulus:

$$E_{Fe} = 1.55 \cdot 10^{11} \frac{N}{m^2}$$

$$E_{Cu} = 4.13 \cdot 10^{10} \frac{N}{m^2}$$

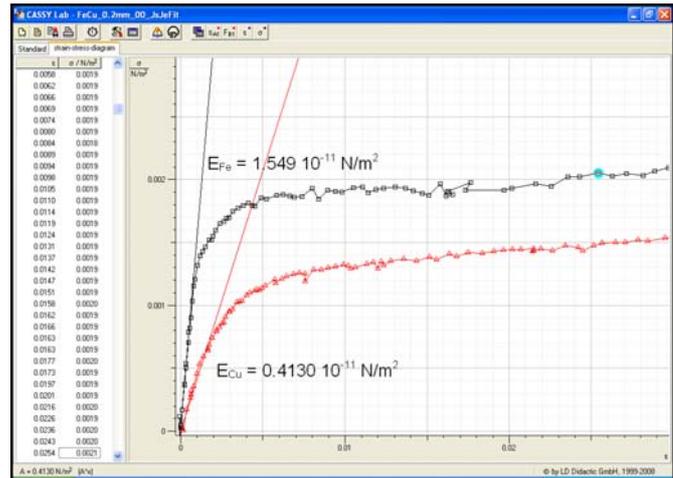


Fig. 10: Measured strain  $\sigma$  as function of expansion  $\epsilon$  of a iron and copper wire. The straight lines correspond to the fit according equation (III) to determine the elastic modulus E.

The comparison of the measurements series of iron and copper shows that the expansion of a wire obeys the Hooke's law, i.e. for small tensile forces the deformations are elastic (indicator of the Force Sensor S does return to zero when releasing the wheel of the Rotary Motion Sensor S). Beyond the elasticity limit the deformations become irreversible (indicator of the force Sensor S does not return to zero). The wire starts to flow. A further increase leads to the breaking point.

**Supplementary information**

The experiment can be performed with different materials. Fig. 11 shows a summary of the results of a copper wire, iron wire and constantan wire with a diameter of 0.2 mm.

*Hint: To create a plot with different measurements various measurement files are loaded to an existing measurement file.*

Fig. 11: Comparison of measured strain  $\sigma$  as function of expansion  $\epsilon$  for copper (red), iron (black) and constantan (blue) wire.

