**Atomic and nuclear physics**

**X-ray physics**

**X-ray tomography**

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**Objects of the experiment**

- Understanding the material absorption of the X-rays.
- Understanding the 3D-image reconstruction used for the computed tomography.
- Device adjustment and quantification of the picture resolution restrictions.

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**Principles**

The initial intensity \( I_0 \) of an X-ray beam is reduced after passing through a homogeneous material according to the Lambert law

\[
I_s = I_0 e^{-\mu s}, \tag{1}
\]

Here is \( I_s \) the resulting beam intensity, \( s \) the length of the way through the material and \( \mu \) the material absorption coefficient. The latter has the physical dimension 1/Length and is material specific, depending a.o. on the material density, atomic number and the energy (wavelength) of the beam. Absorption coefficient reveals therefore material distribution, knowledge of which is of a great importance in medicine and other fields.

For inhomogeneous materials the constant absorption coefficient \( \mu \) is replaced by \( \mu(s) \) and the exponent in eq. (1) becomes an integral

\[
\ln \frac{I_s}{I_0} = - \int_0^s \mu(s') ds'. \tag{2}
\]

For a general absorption coefficient \( \mu = \mu(x,y,z) \), given as a function on the three dimensional space, the integral in eq. (2) is written as a path integral

\[
\ln \frac{I_s}{I_0} = - \int_{(x,y)} \mu(x,y,z) ds'. \tag{3}
\]

along the straight beam path. For a fixed plane \( \Sigma \) (in the three dimensional space) parametrized by a coordinate system \( (x',y') \), any line is given by the slope \( m \) and the \( y' \)-intersect \( b \). For a function \( \mu(x',y') \) on the plane \( \Sigma \) the integral along the line given by \( (m,b) \) is a functional assignment

\[
(m,b) \mapsto \int_{m,b} \mu(x',mx+b) dx'. \tag{4}
\]

called the Radon transformation. According to the Radon’s theorem (J. Radon (1917)), the function \( \mu(x',y') \) can be reconstructed if function’s integral is known for all \( (m,b) \).

Computed tomography (CT) is a direct application of Radon’s theorem. According to eq. (3) a line integral of the absorption coefficient \( \mu \) along the beam path in a sectional plane of a body is given by the logarithm of intensity fraction. Measuring therefore intensity fraction for many lines, absorption coefficient \( \mu \) can be reconstructed. Its graphical representation reveals the sectional plane compound structure. Substance identification with application to medicine is studied in experiments P6.3.8.3-4 and P6.3.8.13-14. A CT measurement scheme is shown in fig. 1. Intensity fraction measurement for many lines is realized by the rotation of the body.

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**Measurement with the computed tomography module**

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**Fig. 1:** A schematic representation of a CT-scan. Different colors mark different values of \( \mu \) in the sectional plane \( \Sigma \).

Measurement time restricts the number of rotation angles. A high-pass filter is therefore applied to measured data before reconstructing \( \mu \). A high-pass filter highlights boundaries of different materials (like bones and organs in a human body) as it amplifies the contrasts. It reduces therefore errors caused by the finite number of measurements. This procedure is called the **filtered Radon back transformation** and is the key of the CT algorithm.

The number of rotations and the used high-pass filter influence image quality and its resolution. The resolution is also restricted by the pixel size of the detecting device. A further source of image quality error is the continuous X-ray spectrum. Eq. (1) holds only for the monochromatic beam. Passing through material the X-ray beam spectrum is shifted to higher energies. This effect is called beam hardening and is studied in experiments P6.3.8.4 and P6.3.8.14. Absorption coefficient values in the whole body are obtained by combining measured data for different sectional planes. In this case the smallest image unit is called a voxel. Graphical representation of \( \mu \) is the end result of the computed tomography. Details of the graphical representation are studied in experiments P6.3.8.5 and P6.3.8.15.

The 1979s Nobel Prize in Physiology or Medicine was awarded to Allan M. Cormack and Godfrey N. Hounsfield for their development and application of computed tomography.
In this experiment a CT scan of a phantom is done. The phantom is a test block for the computed tomography consisting of several layers for the valuation of the highly and lowly contrast resolution. Students learn device adjustment, sources of errors and resolution restrictions. A quantitative measurement of hole diameter is done to quantify CT’s resolution power.

**Setup**

- Insert the Au tube in the X-ray apparatus.
- Mount the goniometer in the X-ray experiment chamber and move it to the right side. Fix the goniometer using the two screws in its lower part.
- Put the computed tomography module on the right side of the X-ray apparatus and connect the latter to the computed tomography module using a USB cable.
- Connect the computed tomography module to a PC using a USB cable.

**Camera adjustment**

*Note*

Camera adjustment must be performed prior to the measurement.

- Turn on the PC, the X-ray apparatus and the computed tomography module.
- Start the CT software and find the expandable tabs on the left.
- Choose the video device (fig.2: 3 and 4) and turn it on (fig. 2: 1) while X-ray beam and high-voltage are off (fig. 3: 2).
- Identify defective pixels and correct (fig. 2: 6 and 7).
- Turn on the X-rays (fig. 3: 1).
- Record a white image averaging over app. 10 images and use this as a reference image (fig. 2: 8 and 9).
- Turn off the X-rays (fig. 3: 2).

**Rotary axis adjustment**

*Note*

Rotary axis adjustment must be performed after every new device setup.

- Fasten a rubber band on the tip of the PMMA arms of the object holder to keep the empty holder stable.
- Mount the object holder into the goniometer and close the X-ray apparatus.
- Turn on the X-rays.

In the now visible X-ray image of the object holder, the prolonged rotation axis should appear horizontal in the middle of the image and coincide with the red line. In case of clearly seen misalignment turn off the X-ray radiation and make a rough camera adjustment. Use the screws around the camera inside the CT module to tilt and roll. The two screws on the left hand side of the optical axis must be rotated in the same amount. Close the computed tomography module and turn on the X-rays and the camera. Carry out a measurement keeping all settings as shown in figs. 2 – 4 by clicking on the button.

After the measurement the X-rays are switched off automatically. Perform the following steps:

- Change to the 2D view by clicking on the button in the upper toolbar.
- Move the slider in the bottom to the left until a sectional plane close to the rubber band is shown on the screen (compare fig. 6).
- Vary the rotary axis value (fig. 5: 1) until the double contours are roughly minimal (fig. 6: b).
- Press “1st position” button (fig. 5: 2) and wait until the software optimizes the position.
- Move the slider in the bottom to the right until a sectional plane close to the object holder’s bottom plane is shown on the screen.
- Vary the rotary axis value (fig. 5: 1) until the double contours are minimal (fig. 6: b).
- Press “2nd position” button (fig. 5: 3) and wait until the software optimizes the position.
- Turn off the X-rays and adjust camera position using the screws in a way the CT software suggest (fig. 5: 4, 5 and 6).
Carrying out the experiment

- Carry out the upper steps for the camera and the rotary axis adjustment.
- Increase the number of projections (fig. 4: 1) to 360 or higher depending on the graphic card.
- Unmount the object holder from the goniometer and mount the LEGO® adapter instead.
- Mount the phantom onto the LEGE® adapter and carry out a measurement keeping all other settings as shown in figs. 2 – 4.

During the measurement, the image reconstruction can be observed in 2D as well as in 3D mode. Use the 2D and 3D buttons in the upper toolbar to change modes. The image is gradually getting better as more and more angular steps are done and taken into account in the Radon transformation.

In 2D mode image representation can be changed between “Projection”, “Log projection” and “Filtered log projection” (fig. 7: 1). In this way we can visualize the difference between the standard X-ray image and the filtered projections used for the CT image reconstruction.

Evaluation

(a) General graphical effects:

The main CT software graphical effects are briefly listed below. The detailed analysis of all graphical effects is studied in experiments P6.3.8.5 and P6.3.8.15. A detailed description of the software is available in the instruction sheet. Please see the manual for catalogue number 554821

a.1) 2D and 3D modes

Use the 2D and the 3D buttons in the upper toolbar to switch the modes. In 2D mode it can be changed between different sectional planes. Use the slider in the bottom. Change displays between “Projection”, “Log projection” and “Filtered log projection” (fig. 7: 1).

a.2) Arbitrary sectional plane

Use the button in the upper toolbar while in 3D mode to change the representation between the selected sectional plane or the selected sectional plane and the rest of the scanned object behind it. If the button is active, the sectional plane shown on the screen can be chosen for radiographic evaluations by clicking on the button.

a.3) Colour spectrum

Use the button in the upper toolbar different colour spectra can be chosen. Different colours highlight different absorption coefficient values. Some of the colour schemes are better suited to colour blind users.

a.4) Switching light on

Use the button to turn on a virtual light source, which amplifies the CT-scan’s 3D impression. Expand the “Light Parameters” (fig. 8) to change settings.

a.5) Intensity and transparency

In the “Intensity and Transparency” tab (fig. 9) different representation options can be set up. The upper slider (fig.9: 1) changes the image intensity while the lower slider (fig.9: 2) changes the transparency.

a.6) Stereoscopic view

An even stronger 3D effect is achieved by activating the stereoscopic view with the button. The image in this mode must be viewed through the anaglyph (red-cyan) glasses.
(b) Resolution power measurement

- Change to the 2D view by clicking on the 2D button in the upper toolbar.
- Find the phantom’s sectional plane containing holes of different sizes. Use slider in the bottom of the screen.
- Choose the sectional plane for evaluations by clicking on the button.
- Zoom into the desired area using the button in the context menu by clicking on the right mouse button.
- Use the button in the context menu to create a line profile through the holes.
- Copy and extract the line profile values. Use the button in the context menu.
- Change contrast and intensity using the slider in the radiography tab.

**Measuring example**

a) Graphical effects:

Fig. 10 shows how different material can clearly be distinguished by color or brightness respectively. The color gradient in the direction of the rotary axis is due to the hardening effects.

In fig. 11 only two of four hold sets are resolved. This is due to resolution restrictions, which are discussed below. Using the X-Ray image sensor, the two here missing hole sets can be resolved as demonstrated in experiment P6.3.8.12.

The hardening effects are responsible for two colors in the phantom’s 3D view (fig. 12, left). The phantom appears to be covered by an envelope made out of a soft material.

Fig. 13 shows the stereographic representation. The image must be viewed through the anaglyph (red-cyan) glasses.
b) Resolution power measurement:

As seen from fig. 11, only two sets of holes can be seen. Fig. 14 shows two examples of line profile measurements. The x-axis in the diagrams are scaled differently. The hole diameters obtained from the line profiles are given in tab. 1.

![Resolution power measurement](image)

Fig. 14: Resolution power measurement: (left) Line profiles along two the resolved hole sets, (right) the corresponding diagrams. X-axis of the diagrams is differently scaled.

<table>
<thead>
<tr>
<th>Original Size</th>
<th>Size 1</th>
<th>Size 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>mm</td>
<td>1,60</td>
<td>0,80</td>
</tr>
<tr>
<td>CT Result</td>
<td>1,42</td>
<td>0,95</td>
</tr>
</tbody>
</table>

Tab. 1: Hole diameters obtained from the line profiles shown in fig. 14.

Line profiles of a big and a small hole plotted in the same coordinate system are shown in fig. 15. Absorption coefficient in the line profile of the small hole remains clearly larger than 0. Thus the resolution power is at most of app. 1 mm.

Image resolution restrictions due to CT geometrical setup can be estimated. The X-ray is radiated from an anode with the initial ray size of app. 1,5 mm. The rays reach a point at the object’s position in the experiment chamber under an angle. The distance from the anode to the object is app. 32 cm. The distance from the object to the screen is app. 8 cm. The point at the object’s position is projected on the screen with at least a size of

\[ x = \frac{1,5 \text{ mm}}{32 \text{ cm}} \times 8 \text{ cm} = 0,375 \text{ mm}. \]  

(5)

If camera resolution is set to 500x500 px², which is distributed on the screen with the diameter of 15 cm, then a pixel is covering a square on the screen with the side length of 0,3 mm. This is clearly smaller than the geometric resolution restriction given by eq. (5).

The fact, that the holes smaller than 0,8 mm are not resolved, is explained by the fluorescent screen. The latter scatters the radiated light, reducing therefore the image sharpness. Also the number of projections used for the image reconstruction has an impact on the resolution.

Experiment P6.3.8.12 show the impact of the fluorescent screen and the number of projection on the CT’s resolution using the X-ray image sensor.

c) Distance measurement:

A distance can be measured using line profile between peaks (fig. 17). In this case the length is extracted using the line profile values. The distance measured here is 5,1 mm.

![Distance measurement](image)

Fig. 17: A distance measurement using the line profile between two peaks.

Alternatively choose a start and an end point directly in 3D mode. The distance between the points is shown on the screen (fig. 18).

![Distance measurement 3D](image)

Fig. 18: Distance measurement in 3D mode. The distance is measured in the sectional plane shown on the screen.