# Optics

Laser Optics

*He-Ne Laser*

*LD Physics Leaflets*

**P5.8.1.3**

# Determining the Beam Diameter Inside the Resonator

## Experiment Objectives

- Determining the beam diameter at different positions inside the laser resonator
- Representing the beam path in the laser resonator

## Principles

The helium-neon laser is among the most common lasers. In experiment P5.8.1.1, a helium-neon laser is assembled using individual components.

In experiment P5.8.1.3, the beam path inside the resonator is studied, i.e. the magnitude of the beam radius as a function of position along the direction of propagation. The beam path can also be well-described here by Gaussian shaped beams.

### Confocal resonator

The representation of a confocal resonator, where the curvature radiuses $R_1$ and $R_2$ of resonator mirror exactly match the mirror spacing $L$: $R_1 = R_2 = R = L$, is particularly simple.
(Positive curvature radiuses mean that the mirrors are concave in relation to the resonator interior, as in Fig. 1.)

The transverse intensity distribution of the fundamental mode, $TEM_{00}$, is described by a Gaussian distribution (see P5.8.1.2):

$$I(r) = I_0 \cdot e^{-\frac{2r^2}{\omega^2}} \qquad (1)$$

Here, $I_0$ is the maximum intensity and $r$ is the distance to the optical axis. The distance from the optical axis is defined as the beam radius $\omega$ (beam diameter: $2\omega$), at which the intensity has dropped to $I_0/e^2$.

The following applies for the beam path in a confocal resonator:

$$\omega(z) = \omega_0 \cdot \sqrt{1+\left(\frac{2z}{R}\right)^2} \qquad (2)$$

Here, $z$ is the distance from the beam waist.

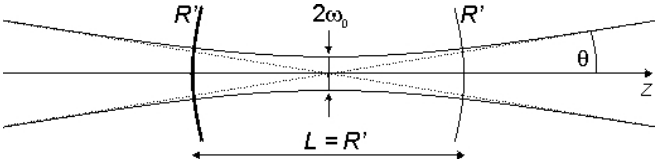

Fig. 1: Beam path in a confocal resonator (mirror radius $R'$, mirror spacing $L = R'$)

The beam waist is in the middle of the resonator (see Figure 1); the beam radius is at its minimum value here

$$\omega_0 = \sqrt{\frac{\lambda R}{2\pi}} \qquad (3)$$

with wavelength $\lambda$.

Therefore, the beam path in confocal resonators only depends on the mirror radius $R$ of the resonator mirror and the laser's wavelength $\lambda$.

For long distances ($z >> L$), the beam diameter decreases following an approx. linear law. Here, the beam path can be described by the beam divergence $\theta$.

$$\theta = \sqrt{\frac{2\lambda}{\pi \cdot R}} \qquad (4)$$

### Arbitrary stable resonator with spherical mirrors

For stable resonators with arbitrary spherical mirrors $R_1 \neq R_2$ and arbitrary distances $L$, corresponding formulae are obtained for the beam path $\omega(z)$ and the beam waist radius $\omega_0$. For this purpose, the resonator is embedded in the phase surface of a suitable confocal resonator with mirror radius $R'$ (see Figure 2). The curvature radiuses $R_1 \neq R_2$ of both mirrors coincide with the curvature radiuses of the phase surfaces of the confocal resonator at positions $z_1$ and $z_2$.

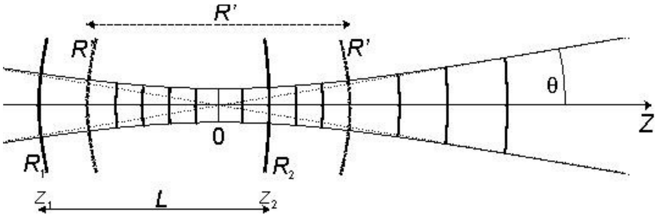

Fig. 2: Beam path for a resonator with $R_1 \neq R_2 \neq$ L

The mirror radius $R'$ of the relevant confocal resonator is determined by the following quotient:

$$R'^2 = L^2 \frac{4g_1g_2(1-g_1g_2)}{(g_1+g_2-2g_1g_2)^2} \qquad (5)$$

By substituting $R'$ in equations (3) and (4), the beam waist diameter $\omega_0$ of the resonator and the beam divergence $\theta$ are obtained.

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Distances $z_1$ and $z_2$ from the spherical mirrors $R_1 \neq R_2$ to the beam waist are obtained using:

$$z_1 = -L\frac{g_2(1-g_1)}{g_1+g_2-2g_1g_2} \qquad (6)$$

$$z_2 = L\frac{g_1(1-g_2)}{g_1+g_2-2g_1g_2} \qquad (7)$$

where $g_1 = 1-\frac{L}{R_1}$, $g_2 = 1-\frac{L}{R_2}$.

The resonator is stable when the following applies:

$$0 \leq g_1g_2 \leq 1 \qquad (8)$$

(Stability criterion for stable resonators, see experiment P5.8.1.5).

**Plano-concave resonator**

If a plane mirror is used, then the mirror radius is infinite (see Figure 3). When $R_1 \rightarrow \infty$, $g_1 = 1$

As a result, equations (5) through (7) can be simplified as

$$R^2 = 4L(R_2 - L) \qquad (9)$$

$$z_1 = 0\text{ m} \qquad (10)$$

$$z_2 = L \qquad (11)$$

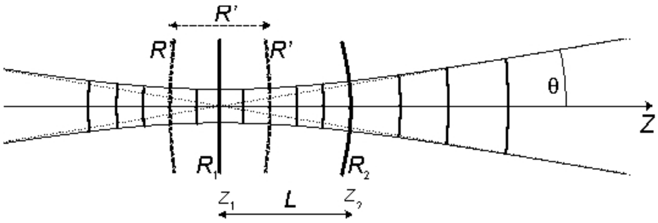

Fig. 3: Beam path for a confocal resonator with $R_1 \rightarrow \infty$ and $R_2 \neq$ L

In this experiment, the beam diameter is measured at different points inside the resonator by using a precision vernier calliper. In order to accurately measure, the vernier calliper is adjusted to a fixed value and the exact position at which the laser process terminates is searched for. The width reading is proportional to the beam diameter. The measurement is repeated for different set widths on both sides of the laser tube. The measurement results are represented graphically and compared with the theoretical values.

## Apparatus

| | |
|---|---|
| 1 Basic Set "He-Ne Laser" | 471 810 |
| 1 Optical bench, 2 m, standard cross section | 460 33 |
| 1 Precision vernier calliper | 311 54 |
| 1 Screen | 441 531 |
| *Additionally recommended:* | |
| 1 Laser mirror, HR, R = -1000 m | 470 103 |
| Adjustment goggles for He-Ne laser | 471 828 |

## Safety Notes

Important: Make sure you also follow the instructions provided with the equipment!

The installed He-Ne laser complies with the Class 3B regulations according to DIN 60825-1 "Safety of Laser Products". Lasers belonging to Class 3B are potentially dangerous if a direct or mirror-reflected beam reaches the unprotected eye (directly looking at the beam).

- Do not look at the direct or reflected laser beam!
- Avoid unintentional mirror-reflections (e.g. through watches, jewellery, tools)!
- Block all laser beams by placing an absorbing or diffuse scattering material at the end of the purpose-related beam path.
- Wear laser adjustment goggles (471 828) if necessary.

Laser tubes require voltages >12 kV to ignite the gas discharge and contact-hazardous voltages of up to 2.5 kV for operation.

- The connection to the supply device should only be established through the high voltage plugs.
- Wiring and changes in the experiment setup should only be carried out when the supply device is switched off.

The supply device should only be switched on when the circuit is completed.

## *Preliminary remarks*

*The experiment only succeeds when the setup is thoroughly adjusted and all optical surfaces are free of impurities. Cleaning a precision optics system always represents a risk for the surface. In order to reduce the need to clean the optics as much as possible, they should be preserved in their original packing or they should be covered with a protective cover and placed in their support when they are not in use.*

*During the experiment, take measures to avoid the damage of mirror surfaces (including the rear side of the output mirror) and the Brewster window of the laser tube. Do not touch them with your bare hands. Immediately remove fingerprints, oil or water stains, because the skin acids attack the coating on the glass and permanent stains can be left behind.*

*If cleaning is necessary, it is advisable to use one of the methods recommended in the instruction sheets.*

## Setup

Setup and calibration of the He-Ne laser are described in experiment P5.8.1.1. $L$ = 90 cm is used as the mirror separation distance at first. The laser tube is placed in the middle of the resonator. Setup is shown in Figure 3. The left edge position of the optics riders is given in cm for each element respectively.

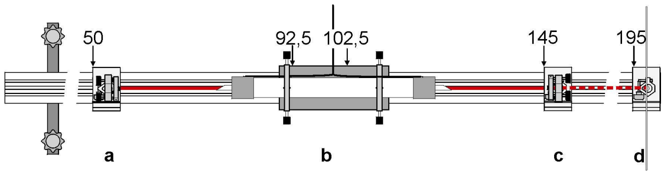

Fig. 4: Experiment setup

- **a** Highly reflecting plane mirror ($R_1 \rightarrow \infty$)
- **b** Laser tube in laser support
- **c** Output mirror OC, $R_2$ = 1000 mm
- **d** Screen

## Method

### Determining the beam radius at the output mirror

- Adjust the vernier calliper to $d$ = 2.0 mm.
- Place the vernier calliper as perpendicular as possible to the beam path approx. at position $x$ = 140 cm in the ray trajectory between the laser tube (**b**) and the output mirror (**c**).
- Carefully move the vernier calliper up and down vertically, while the adjusted slit is approx. at the height of the beam path. When the beam passes exactly through the adjusted slit, laser process repeatedly starts for a short time.
- Reduce the aperture d of the vernier calliper in 0.05 mm increments and check if the laser process starts at $x$ = 140 cm by carefully moving the calliper up and down.
- Write down both the last adjustment d of the vernier calliper for which the laser process starts at $x$ = 140 cm, and its position in the table (see Table 1).

### Determining the beam radius at other positions

- Reduce the aperture of the vernier calliper by 0.05 mm. Starting approx. at $x$ = 130 cm move towards greater $x$ values, testing if the laser process still starts. Write down the last position of x that still started the laser process, as well as the slit width $d$.
- Repeat the measurement for other adjustments $d$ of the vernier calliper, and between the highly reflecting mirror (**a**) and the laser tube (**b**).

If necessary, repeat the measurements for other, noticeable or different, mirror separations or combinations of mirror radiuses.

We obtain the possible mirror separations $L$ from the stability condition (equation 8):

$R_1 \rightarrow \infty$, $R_2$ = 1000 mm: $0 \le L \le 1000\,\mathrm{mm}$

$R_1 = R_2$ = 1000 mm: $0 \le L \le 2000\,\mathrm{mm}$

## Measuring example and analysis

### Plano-concave resonator

Table 1 shows a measuring example for the parameters $R_1 \rightarrow \infty$, $R_2$ = 1000 mm, and $L$ = 90 cm. The distance to the beam waist $z$ is calculated on the basis of $x$ in order to be able to compare it with the theoretical values; $z$ is at the plane mirror position in the setup with a plane mirror. To properly compare, 55 cm (position of the plane mirror) are subtracted from each $z$ value: $z = x - 55$ cm. The adjustment of the vernier calliper $d$ corresponds to the beam diameter $2\omega_m$. The beam radius $\omega_m$ results from $\omega_m = d / 2$.

| $d$ / mm | $x$ / cm | $z$ / cm | $\omega_m$ / mm |
|---|---|---|---|
| 1.60 | 140 | 85 | 0.800 |
| 1.55 | 135.5 | 80.5 | 0.775 |
| 1.50 | 132.5 | 77.5 | 0.750 |
| 1.45 | 129 | 74 | 0.725 |
| 1.40 | 125.5 | 70.5 | 0.725 |
| 1.35 | 122.5 | 67.5 | 0.700 |
| 0.85 | 76 | 21 | 0.675 |
| 0.80 | 71.5 | 16.5 | 0.425 |
| 0.75 | 68 | 13 | 0.400 |
| 0.70 | 65 | 10 | 0.375 |
| 0.65 | 61 | 6 | 0.350 |

Tab. 1: Measuring example for $R_1 \rightarrow \infty$, $R_2$ = 1000 mm and $L$ = 90 cm

In Figure 5, the measured values are shown together with the theoretical expected curve. The measured values represent the relative path well. However, they are located systematically above the expected curve.

The observed divergence is due to the selected measurement method: An additional slit is introduced in the resonator by the vernier calliper. When the additional losses produced by the vernier calliper are so high that the laser process collapses, the slit width exactly matches the beam radius. In general, the possible additional losses can be higher, in the case of larger slit width, or lower, if the corresponding slit width is smaller. That's why the measurement only produces a value that is proportional to the beam radius. The proportionality factor depends on the selected mirrors, separations and calibrations, among other things, and is generally different for each setup.

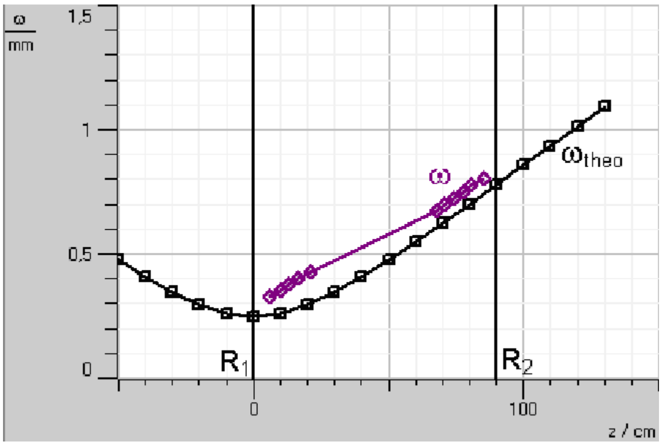

Fig. 5: Beam path in the resonator, measured values (◇) and theoretical values (□) for $R_1$ = 1000 mm, $R_2$ = 1000 mm and $L$ = 90 cm

### Biconcave resonator

Table 2 shows a measuring example for $R_1$= $R_2$ = 1000 mm and $L$ = 90 cm. Here, the beam waist is in the middle of the resonator; so for the calculation, 100 cm are subtracted from $z$ respectively: $z = x - 100$ cm.

| $d$ / mm | $x$ / cm | $z$ / cm | $\omega'$ / mm |
|---|---|---|---|
| 0.75 | 144 | 44 | 0.375 |
| 0.725 | 126 | 26 | 0.363 |
| 0.725 | 73 | 27 | 0.363 |

Tab. 2: Measuring example for $R_1$ = $R_2$ = 1000 mm and $L$ = 90 cm

Figure 6 shows the measured values together with the theoretical expected curve trace. The variation of the beam diameter in the resonator is small; therefore, this corresponds with theory.

The comparison of the curve paths in Figures 4 and 5 shows that the variation of a parameter (here the mirror radius $R_1$) produces a noticeable change in the beam path.

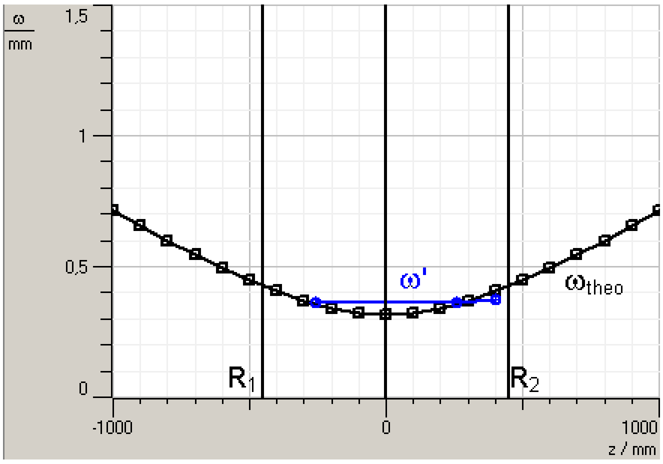

Fig. 6: Beam path in the resonator, measured values (○) and theoretical values (□) for $R_1$ = 1000 mm, $R_2$ = 1000 mm and $L$ = 90 cm