Electricity

DC and AC circuits

Electrical Work and Power

LD Physics Leaflets

P3.6.6.5

Determination of active and reactive power in AC circuits

Experiment Objectives

- Determination of the active and apparent power of an RC series connection
- Determination of the apparent and reactive power with purely inductive or capacitive load

Principles

In an AC circuit with ohmic resistance *R*, inductive resistance X_L and capacitive resistance X_C, the current and the voltage are not in phase, but they have a phase shift φ that depends on the magnitudes of *R*, X_L and X_C.

$$\tan\varphi = \left(X_C - X_L\right)\cdot R \qquad (1)$$

The momentary power *P* then amounts to

$$P = U_0 \sin(\omega t)\cdot I_0 \sin(\omega t + \varphi) \qquad (2)$$

Depending on the magnitude of phase angle φ, *P* is either positive or negative. For positive values, power is taken from the current source; for negative values, it is supplied.

For periodic voltages and currents, consider the average power over a period *T*. This number is called the active power P_W. The active power is the power available to convert electric energy into other forms of energy (e.g. thermal energy).

$$P_W = \frac{1}{T}\int_0^T P\,\mathrm{d}t \qquad (3)$$

Inserting and integrating the equation gives

$$P_W = \frac{1}{2}U_0 I_0 \cos\varphi = U_{eff} I_{eff} \cos\varphi \qquad (4)$$

with the effective values U_{eff} of the alternating voltage and I_{eff} of the alternating current.

The reactive power $P_B = U_{eff} I_{eff} \sin\varphi$ is used to generate electric or magnetic fields in capacitors or coils. It is received by the supply grid if voltage and current have the same sign and fed back if they have different signs. In the middle, the power is worth zero.

The product $U_{eff}\cdot I_{eff}$ is called the apparent power P_S. It is the geometric sum of the reactive power and active power.

$$P_S^2 = P_W^2 + P_B^2$$

The following applies to an electric circuit with an RC series connection:

$$I_{eff} = \frac{U_{eff}}{\sqrt{R^2 + \frac{1}{\omega^2 C^2}}} \qquad (5)$$

So the apparent power $P_S = U_{eff}\cdot I_{eff}$ amounts to:

$$P_S = \frac{U_{eff}^2}{\sqrt{R^2 + \frac{1}{\omega^2 C^2}}} \qquad (6)$$

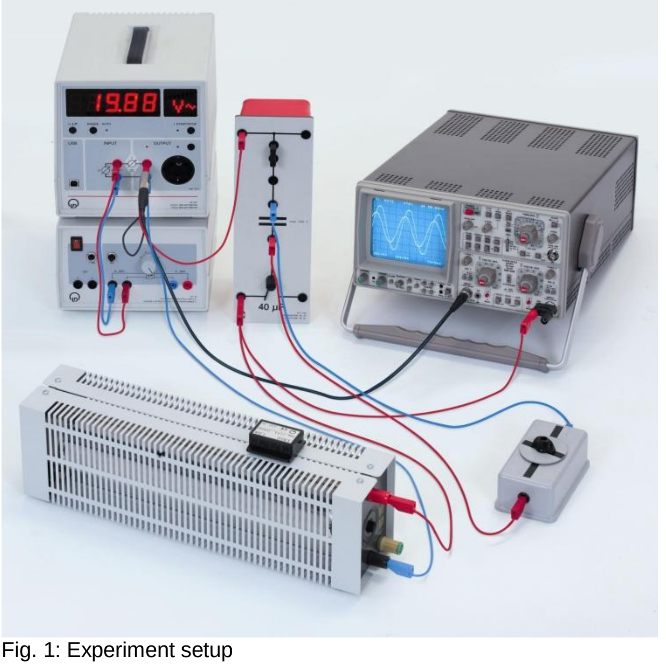

Fig. 1: Experiment setup

Its greatest value is at *R* = 0, and it diminishes monotonously as *R* rises.

The active power heats the resistance R with the current I_{eff}:

$$P_W = I_{eff}^2 \cdot R = U_{eff}^2 \frac{R}{R^2 + \frac{1}{\omega^2 C^2}} \qquad (7)$$

Therefore $P_W = 0$ if $R = 0$. For the resistance

$$R_0 = \frac{1}{\omega C} \qquad (8)$$

P_W is at its maximum, i.e. *R* corresponds to the capacitor's impedance $\frac{1}{\omega C}$.

The first part of the experiment will measure the current I_{eff}, the phase shift φ and the active power P_W as a function of the resistance *R* in an RC series connection with a constant alternating voltage *U*.

The second part of the experiment will measure U_{eff}, I_{eff} and P_W of an AC circuit with coil or capacitor without ohmic resistance. Comparing the apparent and active power shows if the assumption of an "ideal" capacitance or inductance is justified in each case.

CS-1107

Apparatus

1 Joule and Wattmeter 531 831
1 Variable extra low voltage transformer S.........521 35
1 Rheostat 330 Ohm 537 35
1 Capacitor, 40 µF..517 021
1 U-core with yoke..562 11
1 Clamping device...562 12
1 Coil with 1,000 turns......................................562 15
1 Single-pole cut-out switch...............................504 45
1 Two-channel oscilloscope 575 212
1 Screened cable BNC/4 mm............................575 24
1 Adapter BNC/4 mm, 2-pole 575 35
2 Connecting lead 19 A, 50 cm, red/blue, pair....501 45
2 Connecting lead 19 A, 100 cm, red/blue, pair..501 46
1 Connecting lead 19 A, 50 cm, red 500 421

Setup

a) Active and apparent power in an RC series connection

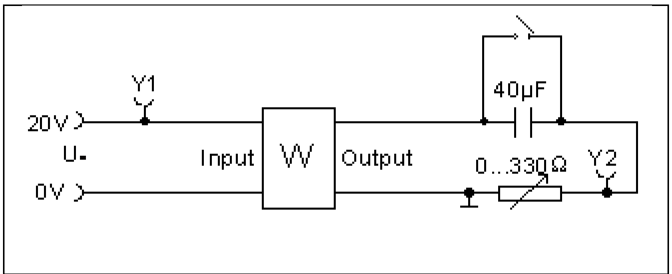

Fig. 2: Experiment setup for Part a)

- The experiment's setup is shown in Fig. 2. The assembly requires the following steps:
- Connect the 0-20 V AC output of the variable extra low voltage transformer S to the input of the joule and wattmeter.
- Connect the capacitor and the rheostat in series to the output of the joule and wattmeter.
- Connect the switch in parallel to the capacitor to bypass the capacitor.
- Connect the oscilloscope's Y1 input to the transformer's output.
- Connect the connections of the oscilloscope's Y2 inputs on both sides of the rheostat to measure the voltage drop at the resistor. In doing so, make sure the ground connection comes between the rheostat and the joule and wattmeter's output (see Fig. 2)!
- Set the rheostat to 330 Ω (maximum).
- Switch the variable extra low voltage transformer S on and set the voltage to 20 V using the dial.

b) Reactive power with capacitive and inductive load

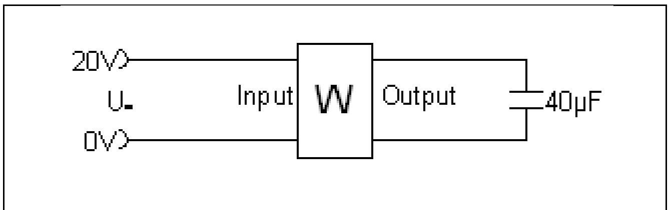

Fig. 3: Experiment setup for Part b)

- The experiment's setup is shown in Fig. 3. The assembly requires the following steps:
- Connect the 0-20 V AC output of the variable extra low voltage transformer S to the input of the joule and wattmeter.
- Connect the capacitor to the output of the joule and wattmeter.
- Switch the power supply on and set the voltage to 20 V using the dial.

Carrying out and Measurement Example

a) Active and apparent power in an RC series connection

- Use the switch to bypass the capacitor.
- On the joule and wattmeter, use the pushbutton *U, I, P* to set the measured value *I* and read the current through the resistor I_R.
- Open the switch and read the current through the resistor and capacitor I_{RC}. For the active power P_W, set the measured value *P* and read the value.
- Use the oscilloscope to determine φ as follows:
  - Overlay both inputs to GD and both tracers to the middle of the screen.
  - Then switch the inputs to AC and the trigger to CH 1.
  - Shift the trigger level and set the time basis so as to set a sine half-wave exactly 9 cm wide.
    Then: 9 cm = 180°.
  - Measure the distance between the zero points for the CH 1 and CH 2 signals. If this distance is x cm, then this path corresponds to a phase shift of

$$\frac{\varphi}{180°} = \frac{x}{9\,\text{cm}} \quad \text{or} \quad \varphi = \frac{x}{\text{cm}} \cdot 20°$$

- Enter the measured values into the table (see Measurement Example 1).
- Incrementally reduce the resistance R and repeat the measurement. In doing so, make sure the resistance does not go to zero (short circuit!).

Measurement Example 1:

No.	I_R / A	I_{RC} / A	φ / °	P_W / W
1	0.071	0.068	16	1.31
2	0.084	0.079	20	1.50
3	0.094	0.087	22	1.62
4	0.113	0.102	26	1.83
5	0.156	0.130	32	2.16
6	0.202	0.152	38	2.32
7	0.304	0.185	50	2.28
8	0.392	0.201	56	2.07
9	0.537	0.215	62	1.73
10	0.712	0.222	64	1.42
11	0.957	0.229	72	1.11

b) Reactive power with capacitive and inductive load

- Switch the power supply on and set the voltage to 20 V using the dial.
- On the joule and wattmeter, read off U_{eff}, I_{eff} and P_W one after the other.
- Switch the power supply off, attach the coil on the U-core with yoke, and replace the capacitor.
- Repeat the experiment.

Measurement Example 2:

	U_{eff}	I_{eff}	P_W
Capacitor	20.1 V	0.240 A	0.00 W
Coil	20.1 V	0.017 A	0.150 W

Evaluation

a) Active and apparent power in an RC series connection

Use the measurement values to calculate:

- Resistance $R = \frac{20\,\mathrm{V}}{I_R}$,
- Apparent power $P_S = 20\,\mathrm{V} \cdot I_{RC}$,
- $\frac{P_W}{P_S}$,
- cos*φ*.

Carry out the evaluation for the data from Measurement Example 1 and fill in the following table.

Evaluation for Measurement Example 1:

No.	R / Ω	P_S / W	P_W / P_S	cos *φ*
1	282	1.36	0.96	0.96
2	238	1.58	0.95	0.94
3	213	1.74	0.93	0.93
4	177	2.04	0.90	0.90
5	128	2.60	0.83	0.85
6	99.0	3.04	0.76	0.79
7	65.8	3.70	0.62	0.64
8	51.0	4.02	0.51	0.56
9	37.2	4.30	0.40	0.47
10	28.1	4.44	0.32	0.44
11	20.9	4.58	0.24	0.31

From equation (4), we have $\frac{P_W}{P_S} = \cos\varphi$. Comparing the values in the table shows the experiment fulfills this. The deviations' increase when R is low is related to the inaccuracy of determining the phase. A 1° reading error at φ = 70° leads to about a 10 % error in determining cos φ, while the same reading error at φ = 20° causes a deviation of only 1 %.

The resistance R_0, at which P_W reaches its maximum, can be determined from a graphic representation of P_W against R (see Fig. 4). From equation (7), a curve is fit to the measured values (in CASSY Lab). The fig. gives a value of R_0 = 85 Ω for the example.

Thus, from equation (8) with frequency f = 50 Hz, we have:

$$C = \frac{1}{2\pi \cdot f \cdot R_0} = 38\,\mu\mathrm{F}$$

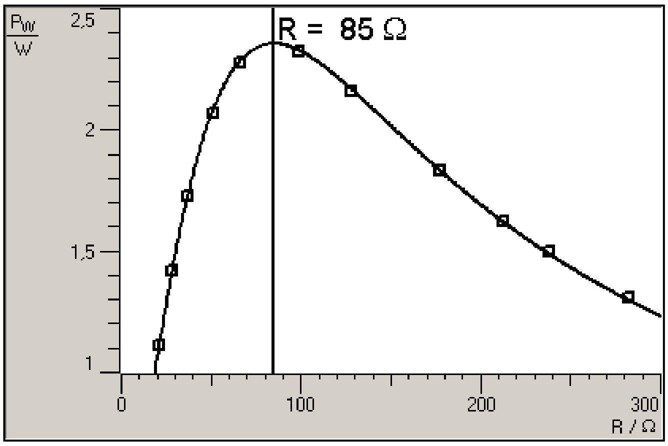

Fig. 4: Dependence of active power on resistance

b) Reactive power with capacitive and inductive load

The apparent power P_S and reactive power P_B are calculated from the measurements:

$$P_S = U_{eff} \cdot I_{eff} \text{ and } P_B = \sqrt{P_w^2 + P_S^2}$$

The values from Measurement Example 2 give:

	P_S / W	P_W / W	P_B / W
Capacitor	4.82	0.00	4.82
Coil	0.34	0.15	0.31

Since the active power is equal to zero with the capacitor, this is pure reactive power.

With the coil, the active power is not zero, so the coil is not an ideal inductor. The coil wire's final ohmic resistance and the magnetic loss in the iron core explain this.

With a coil or core without resistance, P_W = 0 W, as with the capacitor. This can be achieved with superconductive coils, for example.