

Electricity

Electromagnetic induction
Measuring the earth's magnetic field

Measuring the earth's magnetic field with a rotating induction coil and wireless CASSY

Objects of the experiment

- Determination of the components of the earth's magnetic field.
- Determination of the inclination angle of the earth's magnetic field.

Principles

When a circular induction coil with N turns and an area $A = \pi \cdot R^2$ rotates at a constant angular velocity ω in an homogeneous magnetic field B around its diameter d as an axis it is permeated by a magnetic flux

$$\Phi = \pi \cdot R^2 \cdot N \cdot B \cdot \cos(\omega \cdot t) \quad (I)$$

ω : angular velocity

R : radius of the induction coil

N : turns of the induction coil

Equation (I) assumes that the axis of rotation is perpendicular to the magnetic field B . The magnetic field B can be determined from the amplitude of the induced voltage U by

$$U = - \frac{d\Phi}{dt} = \pi \cdot R^2 \cdot N \cdot B \cdot \omega \cdot \sin(\omega \cdot t) \quad (II)$$

Using the revolution time $T = 2 \cdot \pi / \omega$ we find for the peak value of the induced AC voltage:

$$\hat{U} = \frac{2 \cdot \pi^2 \cdot N \cdot R^2}{T} \cdot B = a \cdot B \quad (III)$$

$$a = \frac{2 \cdot \pi^2 \cdot N \cdot R^2}{T} \quad (IV)$$

For a rotation of the coil around the z -direction of a Cartesian coordinate system (Fig. 1) the voltage amplitude

$$U_z = a \cdot \sqrt{B_x^2 + B_y^2} \quad (V)$$

is induced by the earth's magnetic field

$$\vec{B} = \begin{pmatrix} B_x \\ B_y \\ B_z \end{pmatrix} \quad (VI)$$

In this experiment the axis of rotation of the induction coil is aligned in the z -direction of the room. The amplitude of the induced voltage is measured as a function of time with CASSY. From the recorded signals the amplitude and frequency is used to determine the horizontal strength of the earth's magnetic field.

Apparatus

1 Pair of Helmholtz coils.....	555 604
1 Gyroscope axle	348 22
1 Pocket-CASSY 2 Bluetooth.....	524 018
1 Battery Pack.....	524 019
1 Bluetooth Dongle.....	524 0031
1 UIP Sensor S.....	524 0621
1 Universal bosshead.....	666 615
1 CASSYLab 2	524 220
1 Connecting leads 25 cm, pair.....	501 44

Additionally required:

1 PC with Windows XP or up



Fig. 1: Experimental setup.

Setup

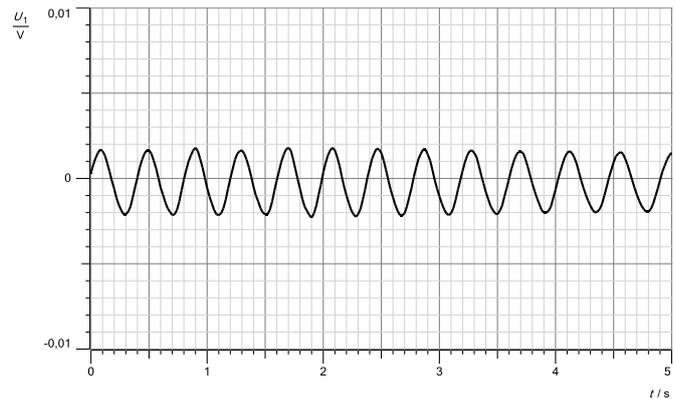
- Connect the Gyroscope axle and one of the Helmholtz coils with the universal bosshead
- Plug together the Battery pack, the Pocket CASSY2 Bluetooth and the UIP Sensor. The Pocket CASSY 2 will blink blue
- Glue one strip of hook-and-loop tape to the Pocket CASSY and the corresponding one to the Helmholtz coil and attach the CASSY (with battery and UIP Sensor) to the Helmholtz coil as shown in Fig 1
- Connect the coil to the UIP Sensors Voltage input with the connecting leads.
- Put the Bluetooth dongle into the PC or use a PC with integrated Bluetooth capabilities.
- Start CASSYLab 2 and make sure the Bluetooth icon is enabled, otherwise see instructions for the Pocket CASSY 2 Bt 524018 on how to switch the stack to the Microsoft stack.
- Make sure the Pocket CASSY 2 is still blinking blue, if it is dark, press its button to wake it up.
- Establish a Bluetooth link between the Pocket CASSY 2 and the computer from within CASSYLab2 by pressing the Bluetooth icon first and after the Pocket CASSY 2 is showing up in that windows, check "utilize".

Carrying out the experiment

- Select the most sensitive voltage range ($\pm 0,1$ V) of the UIP Sensor.
- Set the sampling interval to 20 ms and the measurement time to 5 seconds, "repeating measurement", "Trigger = 0V, rising"
- Rotate the coil by hand around the axis
- Start the measurement of the induced voltage as function of time by pressing the function key F9 or using the button  alternatively.
- **Note:** The measurement stops automatically after 5 s.

Measuring example

In Fig. 2, as an example, a voltage curve is shown.

Fig. 2: Induced voltage U as function of time

Evaluation and results

To determine the components of the earth's magnetic field the amplitude and frequency has to be determined. There are several ways to determine the frequency.

Method 1

- Click right mouse button in the display and choose "Set Marker/Measure Difference" (Fig. 3).
- Click e.g. on a zero voltage position and repeat this at the position after the period T .
- Alt T allows you to display the result of the status line in the display (Fig. 3).

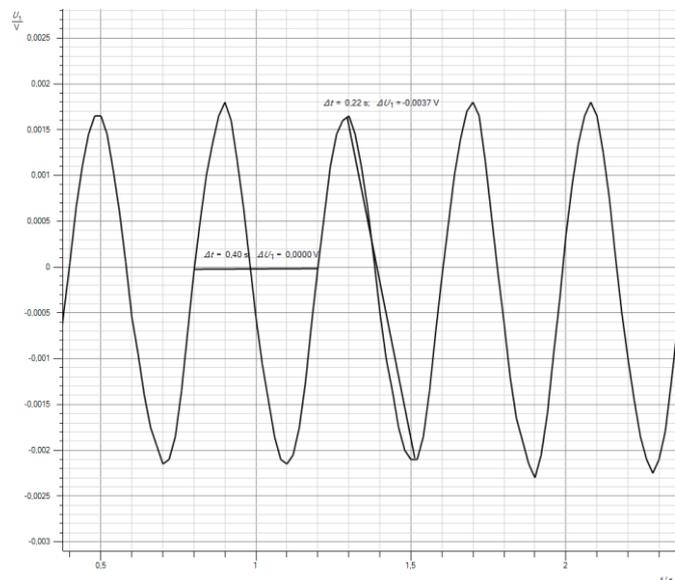


Fig. 3: Determination of the Frequency (Method 1). Compare Fig. 4 and Fig 5.

- This will give the duration of one period. Frequency can be calculated from it.
- To increase the precision, more than one period might be measured..

Method 2

- Go to "Calculator", "FFT" and click "New"
- A tab "Frequency Spectrum" is created
- Adjust values of the FFT Quantity if necessary
- Click right mouse button in the display and choose "Other Evaluations" / "Calculate Peak Center" (Fig. 4).
- Mark the highest peak by dragging the mouse over the experimental data.
- Alt T allows you to display the result of the status line in the display (Fig. 4).
- If the rotation of the coil is slowing down during the measurement, the peak will be broader. Reduce the measurement time accordingly.

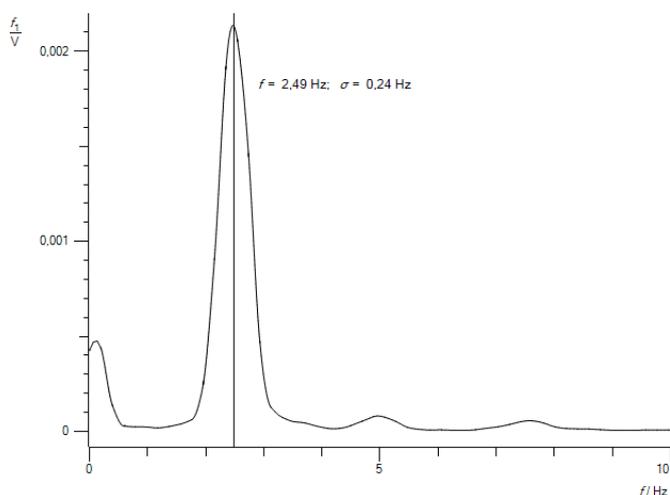


Fig. 4: Determination of the Frequency (Method 1). Compare Fig. 3 and Fig 5.

Together with frequency, we need to determine the amplitude. There are two ways to determine the amplitude of the induced voltage:

Method 1

- Click right mouse button in the display and choose "Set Marker/Measure Difference".
- Click e.g. on a zero voltage position and repeat this at a position of maximum voltage.
- Alt T allows you to display the result of the status line in the display.

Method 2

This method uses the fitting tool to determine the frequency and amplitude.

- Invoke the fitting tool (Fig. 5) by a clicking the right mouse button in the display and choose "Fit Function" / "Free Fit" or use the speed up key Alt F.
- First select the appropriate fit function – here:

$$f(x,A,B,C,D) = A \cdot \sin(360 \cdot B \cdot x + C) \quad (\text{VII})$$

- Enter appropriate estimate values (Starting values) for the fit parameter

- $A = U_0 \approx 1 \text{ mV}$ (amplitude – read from y-axis of the diagram)
- $B \approx 2.5 \text{ Hz}$ (frequency – about 2 Hz)
- $C = 0$ (phase shift – zero due to measure conditions)
- $D = 0$ (additional fit parameter – not used)
- Continue with the button "Continue with Range Marking"
- Alt T allows you to display the result of the fit quickly in the display.

Note

The fit algorithm needs to have good estimate values to perform the fit successfully. For this fit model, i.e. equation (XIV), the start value of the frequency has to be chosen close to the experimental value.

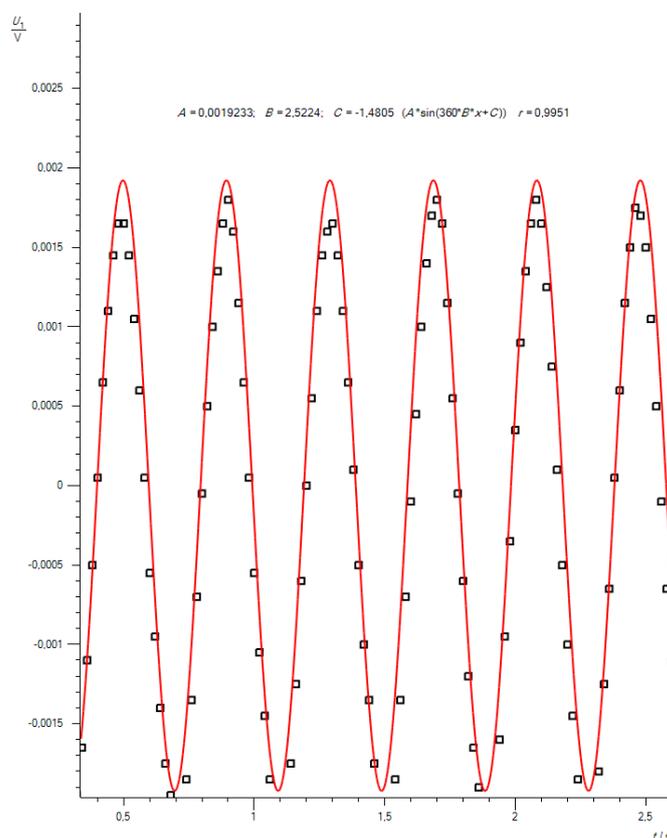


Fig. 5: Induced voltage U_y (black dots) as function of time (y-direction = axis of rotation). The red line corresponds to a fit according to the parameter listed below the voltage curve.

Note: The fitting tool can be invoked by a clicking the right mouse button in the display and choose "Fit Function" / "Free Fit".

Fig. 5. shows a fit to the measured voltage values. Within the error limit the verifies nicely the sinusoidal form. For further data evaluation tools of CASSY Lab see supplementary information.

In the example of figure 5 we got about 1.9 mV amplitude at 2.5 Hz frequency

Result

With the coil parameter $d = 13.5$ cm ($R = 6,75$ cm) and $N = 320$ the strength of the earth's magnetic field can be determined using equation (III) and (IV):

$$a = \frac{2 \cdot \pi^2 \cdot N \cdot R^2}{T} = 72 \text{ m}^2/\text{s}$$

$$B = U/a = 26 \text{ } \mu\text{T}$$

Supplementary information

The main source of error in the experiment is the distortion of the magnetic field due to magnetized steel parts near the conductor loop.

The magnitude of the magnetic field on earth where no inclination occurs (magnetic equator) is about $31.2 \text{ } \mu\text{T}$ and about a factor of 2 larger at the magnetic poles. But due to inclination, the horizontal component is smaller.

In this experiment we measure the absolute size of B without sign, so there is no difference north or south of the equator.