Determining the capacitance of a sphere in front of a metal plate

Objects of the experiments

- Charging a conducting sphere in front of an earthed metal plate by applying a high voltage *U*.
- Measuring the charge *Q* as a function of the high voltage *U*.
- Verifying the proportionality between the charge and the high voltage, and determining the capacitance *C*.
- Determining the capacitance *C* as a function of the distance *s* between the sphere and the metal plate.

Principles

The potential difference *U* between a charged, insulated electrical conductor in free space and a reference point at infinite distance is proportional to the charge *Q* on the conductor:

$$Q = C \cdot U \qquad \text{(I).}$$

The proportionality coefficient *C* is called the capacitance of the charged body.

For example, the capacitance of a sphere of radius *R* in free space is

$$C = 4 \cdot \pi \cdot \varepsilon_0 \cdot R \qquad \text{(II),}$$

$\varepsilon_0 = 8.85 \cdot 10^{-12} \frac{\text{As}}{\text{Vm}}$: permittivity of free space.

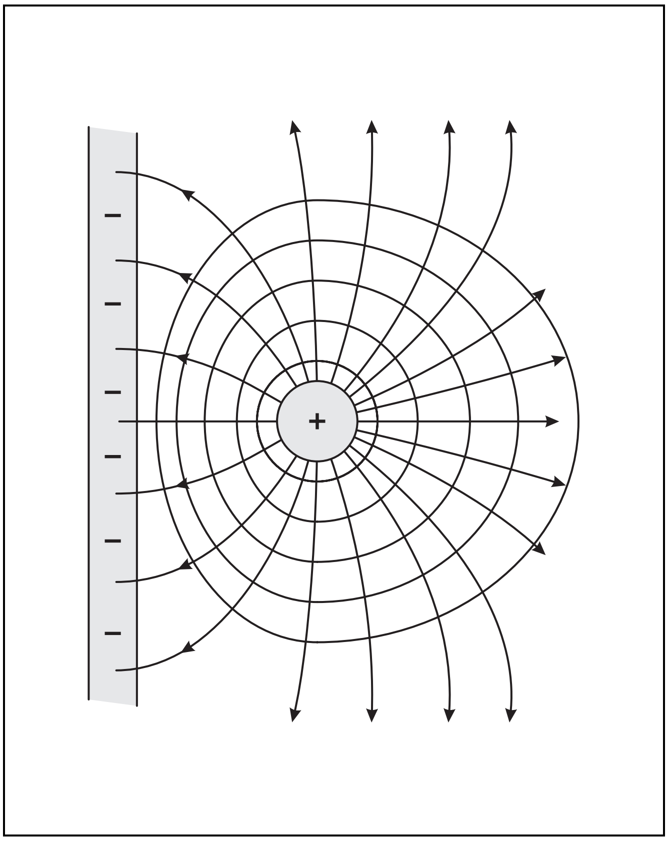
When the sphere is placed near an earthed metal plate, charges of opposite sign are generated on the metal plate due to electrostatic induction. These induced charges lead to a decrease of the potential difference *U*. This effect is the greater, the smaller the distance s is between the sphere and the metal plate. As the charge *Q* remains unchanged, the reduction of the potential difference corresponds, according to Eq. (I), to an enhancement of the capacitance *C*.

In the experiment a conducting sphere of diameter 3.5 cm is placed in front of an earthed metal plate and charged by application of the fixed high voltage *U* = 5 kV at several distances *s*. The increase of the capacitance then becomes manifest in an increase of the charge *Q* on the sphere. In a second series of measurements, the dependence of the charge *Q* on the high voltage *U* is studied at a fixed distance *s* and then compared to Eq. (I).

The charge is measured almost without current by means of an electrometer amplifier operated as a coulombmeter. Any voltmeter may be used to display the output voltage U_A. From the reference capacitance C_A

$$Q = C_A \cdot U_A \qquad \text{(III)}$$

is obtained. For example, at C_A = 10 nF, U_A = 1 V corresponds to the charge *Q* = 10 nAs. If other capacitances are used, other measuring ranges are accessible.



Electric field lines (arrows) and lines of constant potential between a charged sphere and a metal plate

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Apparatus

1 set of three conducting spheres	543 00
1 reflection plate, 50 cm×50 cm	587 66
1 high voltage power supply 10kV	521 70
1 high voltage cable, 1 m	501 05
1 electrometer amplifier	532 14
1 plug-in unit 230 V/12 V~/20 W	562 791
1 Faraday's cup	546 12
1 clamping plug	590 011
1 STE capacitor 1 nF, 630 V	578 25
1 STE capacitor 10 nF, 100 V	578 10
1 connection rod	532 16
1 voltmeter, DC, until U = ±8 V for example	531 100
1 insulated stand rod, 25 cm	590 13
1 stand rod, 47 cm	300 42
3 saddle base	300 11
1 steel tape measure, 2 m	311 77
1 set of 6 croc-clips, polished	501 861
connection rods	

Safety notes

The high voltage power supply 10 kV fulfills the safety requirements for electrical equipment for measurement, control and laboratory. It supplies a non-hazardous contact voltage. Observe the following safety measures.

- Observe the instruction sheet of the high voltage power supply.
- Always make sure that the high voltage power supply is switched off before altering the connections in the experimental setup.
- Set up the experiment so that neither non-insulated parts nor cables and plug can be touched inadvertently.
- Always set the output voltage to zero before switching on the high voltage power supply (turn the knob all the way to the left).
- In order to avoid high-voltage arcing, lay the high voltage cable in such a way that there are no conductive objects near the cable.

Preliminary remark

Carrying out this experiment requires particular care because “leakage currents” through the insulators may cause charge losses and thus considerable measuring errors. Moreover, undesirable effects of electrostatic induction may influence the results.

The experiment must be carried out in a closed, dry room so as to prevent charge losses due to high humidity.

Cleaning the insulator rod which holds the sphere with distilled water is recommended because distilled water is the best solvent of conductive salts on insulators. In addition, the insulator rod should be discharged after every experiment by quickly passing it through a non-blackening flame several times, e.g. of a butane gas burner.

The high voltage power supply and the point of the high voltage cable must be at a sufficient distance from the rest of the experimental setup so as to avoid interference by electrostatic induction.

For the same reason the person performing the experiment – particularly while measuring charges – must keep the connection rod of the electrometer amplifier in his hand to earth himself.

Setup

The experimental setup has two parts. In Fig. 1, the setup for electrostatic charging of the sphere in front of the metal plate is illustrated. Fig. 2 shows the connection of the electrometer amplifier for the charge measurement.

Electrostatic charging of the sphere in front of the earthed metal plate:

– Connect the high voltage cable to the positive pole of the high voltage power supply and the negative pole to earth.
– Mount the insulated stand rod in a saddle base, and put the free point of the high voltage cable **(a)** through the uppermost hole of the insulated stand rod.
– Set the reflection plate up, and connect it to earth.
– Place the insulator rod with the large sphere in front of the reflection plate so that the sphere touches the plate.
– Fix the steel tape measure on the experiment table, and define a suitable zero of the scale for the “right” edge of the saddle base.

Setup for the charge measurement:

– Supply the electrometer amplifier with voltage from the plug-in unit.
– Attach the Faraday‘s cup **(b)** with the clamping plug.
– Attach the capacitor 10 nF **(c)**.
– Use a connection lead to connect the connection rod **(d)** to earth and, if possible, the earth to the earth of the high voltage power supply through a long connection lead.
– Connect the voltmeter to the output.

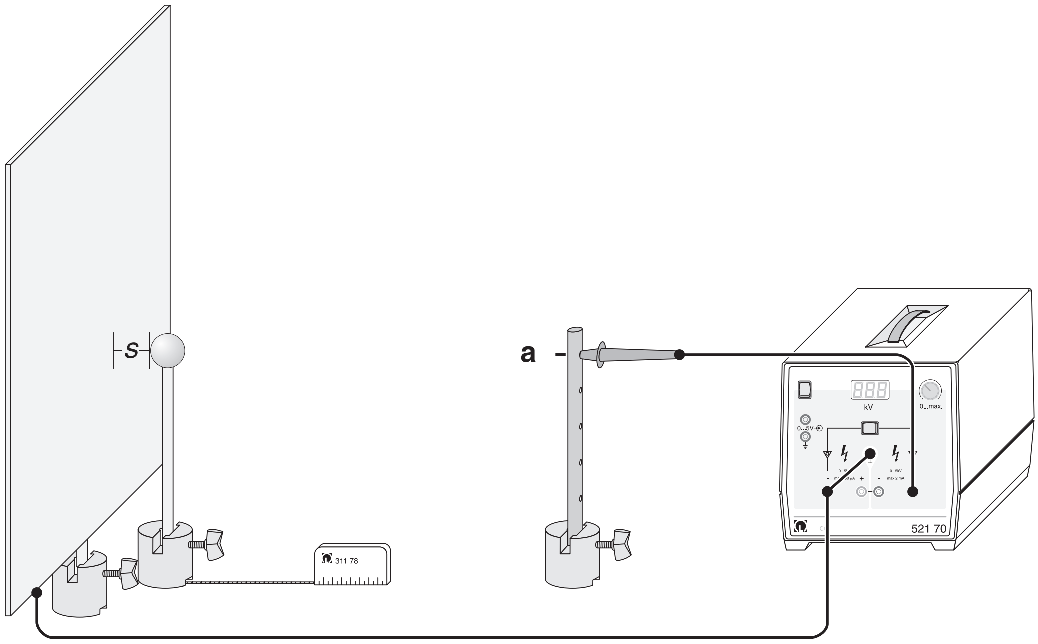


Fig. 1 Experimental setup for electrostatic charging of a sphere in front of an earthed metal plate.

Carrying out the experiment

a) Dependence on the high voltage *U*:

– Choose the distance *s* = 5 cm and switch the high voltage power supply on.
– Set the high voltage U to 1 kV, charge the conducting sphere by touching it with the point of the high voltage cable, and set the high voltage back to zero.

To measure the charge, discharge the Faraday's cup by touching it with the connection rod **(d)**, then take the connection rod in your hand, and move the conducting sphere with the insulator rod to the inside wall of the Faraday's cup (see Fig. 3)

– Increase the high voltage *U* in steps of 1 kV up to 10 kV, and repeat the measurement each time after setting the high voltage back to zero.

b) Dependence on the distance *s*:

– Discharge the conducting sphere at the metal plate.
– Check the zero of the scale for the measurement of the distance, and set the distance *s* to 2 cm.
– Charge the sphere by applying *U* = 5 kV, and measure the charge *Q*.
– Repeat the measurement at larger distances.

Fig. 2 Connection of the electrometer amplifier for the measurement of the charges.

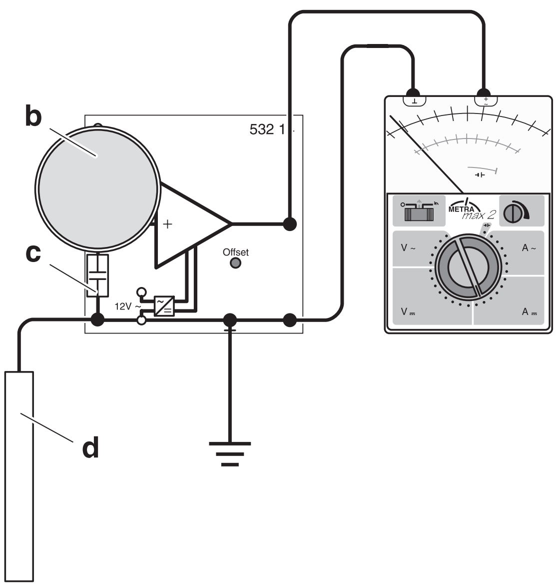
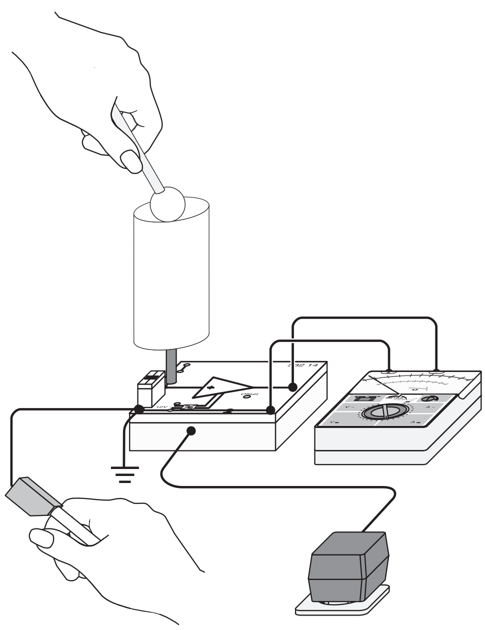


Fig. 3 Measurement of the charge on a conducting sphere.



Measuring example

a) Dependence on the high voltage *U*:

Table 1: The charge *Q* on the conducting sphere as a function of the high voltage *U* (*s* = 5 cm)

$\frac{U}{\text{kV}}$	$\frac{Q}{\text{nAs}}$
1	1.8
2	3.9
3	5.4
4	7.2
5	8.7
6	10.7
7	12.4
8	13.7
9	15.3
10	16

b) Dependence on the distance *s*:

Table 2: The charge *Q* on the conducting sphere as a function of the distance *s* from the metal plate (*U* = 5 kV).

$\frac{s}{\text{cm}}$	$\frac{Q}{\text{nAs}}$
1	12.3
2	10.6
3	9.7
4	9.3
5	8.7
10	8.0
15	7.8
20	7.5

Evaluation and results

a) Dependence on the high voltage *U*:

Fig. 4 is a plot of the measuring values of Table 2. The measuring values lie, within the accuracy of measurement, on the straight line through the origin drawn in the graph. The slope of this line is equal to *C*.

The proportionality $Q \propto U$ is thus confirmed.

b) Dependence on the distance *s*:

Table 3: The capacitance *C* of the conducting sphere as a function of the distance *s* from the metal plate.

$\frac{s}{\text{cm}}$	$\frac{C}{\text{pF}}$
1	2.5
2	2.0
3	1.8
4	1.75
5	1.7
10	1.6
15	1.55
20	1.5

According to Eq. (I), the capacitance *C* of the arrangement of the conducting sphere and the metal plate is obtained after division of the measuring values given in Table 2 by the high voltage *U* = 5 kV (see Table 3).

In Fig. 5, the result is represented graphically. The capacitance *C* turns out to increase nonlinearly with decreasing distance *s*.

Fig. 4 The charge *Q* as a function of the high voltage *U* (distance between the sphere and the metal plate *s* = 5 cm)

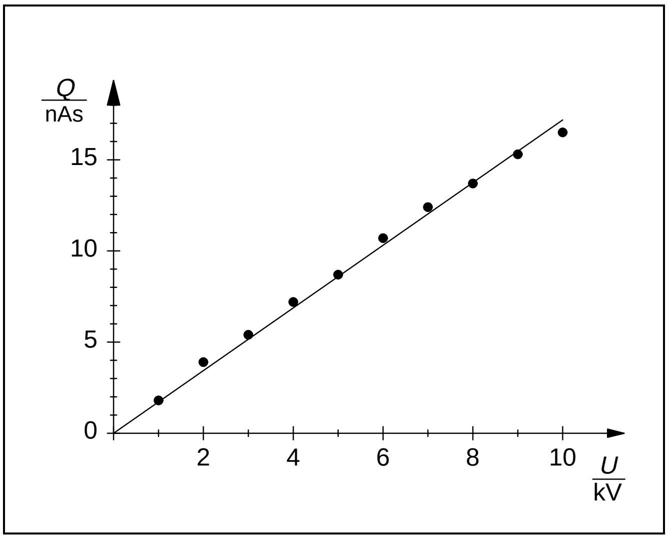


Fig. 5 The capacitance *C* of the conducting sphere as a function of the distance *s* from the metal plate.