

Determining the capacitance of a sphere in free space

Objects of the experiments

- Measuring the charge Q on a conducting sphere in free space as a function of the high voltage U which has been applied to the sphere to charge it.
- Determining the capacitance C of the sphere as a function of its radius R .
- Determining the permittivity of free space ϵ_0 .

Principles

The potential difference U between a charged, insulated electrical conductor in free space and a reference point at infinite distance is proportional to the charge Q on the conductor:

$$Q = C \cdot U \quad (I).$$

The proportionality coefficient C is called the capacitance of the charged body.

For example, the capacitance of a sphere of radius R in free space is

$$C = 4 \cdot \pi \cdot \epsilon_0 \cdot R \quad (II),$$

$\epsilon_0 = 8.85 \cdot 10^{-12} \frac{\text{As}}{\text{Vm}}$: permittivity of free space,

because the potential difference between a charged sphere and a reference point at infinite distance is (cf. Fig. 1)

$$U = \frac{1}{4 \cdot \pi \cdot \epsilon_0} \cdot \frac{Q}{R} \quad (III).$$

In the experiment, several high voltages U are applied to charge conducting spheres of various radii. In each case the charge Q is measured with the aims of confirming the proportionality

$$Q \propto U \quad (IV),$$

determining the capacitance C as a function of the radius R , and confirming the proportionality

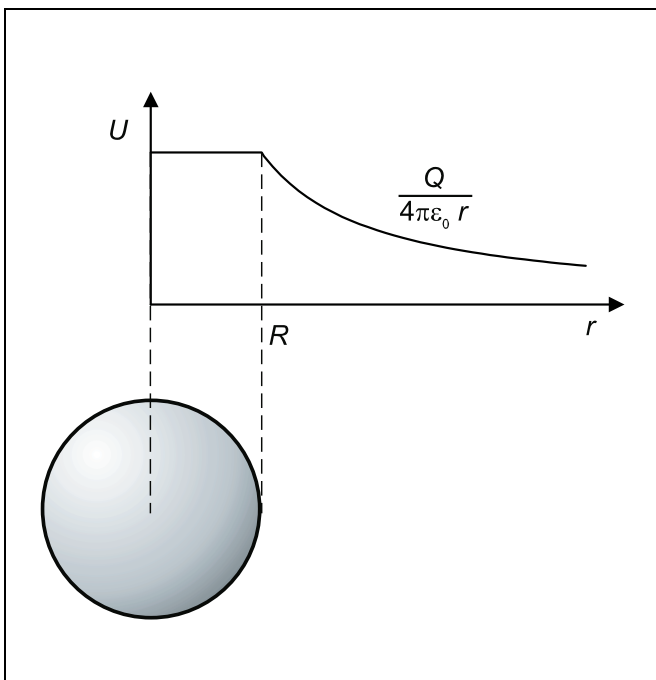
$$C \propto R \quad (V).$$

The charges Q are measured almost without current with an electrometer amplifier operated as a coulombmeter. Any voltmeter may be used to display the output voltage U_A . From the reference capacitance C_A

$$Q = C_A \cdot U_A \quad (VI).$$

is obtained. For example, at $C_A = 10 \text{ nF}$, $U_A = 1 \text{ V}$ corresponds to the charge $Q = 10 \text{ nAs}$. If other capacitances are used, other measuring ranges are accessible.

Fig. 1 The electrical potential of a charged metal sphere of radius R plotted against the distance r from the centre of the sphere.



Apparatus

1 set of 3 conducting spheres	543 00
1 high voltage power supply 10 kV	521 70
1 high voltage cable	501 05
1 electrometer amplifier	532 14
1 plug-in unit 230 V/12 V~/20 W	562 791
1 Faraday's cup	546 12
1 clamping plug	590 011
1 STE capacitor 1 nF, 630 V	578 25
1 STE capacitor 10 nF, 100 V	578 10
1 connecting rod	532 16
1 voltmeter, DC, until $U = \pm 8 \text{ V}$, for example	531 100
1 insulated stand rod, 25 cm	590 13
2 saddle bases	300 11
connection leads	

Preliminary remark

Carrying out this experiment requires particular care because "leakage currents" through the insulators may cause charge losses and thus considerable measuring errors. Moreover, undesirable effects of electrostatic induction may influence the results.

The experiment must be carried out in a closed, dry room so as to prevent charge losses due to high humidity.

Cleaning the insulator rod which holds the spheres with distilled water is recommended because distilled water is the best solvent of conductive salts on insulators. In addition, the insulator rod should be discharged after every experiment by passing it several times quickly through a non-blackening flame – for example of a butane gas burner.

The high voltage power supply and the point of the high voltage cable must be at a sufficient distance from the rest of the experimental setup so as to avoid interference by electrostatic induction.

For the same reason the experimenter – particularly while measuring charges – must keep the connection rod of the electrometer amplifier in his hand to earth himself.

Safety notes

The high voltage power supply 10 kV fulfills the safety requirements for electrical equipment for measurement, control and laboratory. It supplies a non-hazardous contact voltage. Observe the following safety measures.

- Observe the instruction sheet of the high voltage power supply.
- Always make certain that the high voltage power supply is switched off before altering the connections in the experimental setup.
- Set up the experiment so that neither non-insulated parts nor cables and plug can be touched inadvertently.
- Always set the output voltage to zero before switching on the high voltage power supply (turn the knob all the way to the left).
- In order to avoid high-voltage arcing, lay the high voltage cable in such a way that there are no conductive objects near the cable.

Setup

The experimental setup has two parts. In Fig. 2, the setup for electrostatic charging of the spheres is illustrated. Fig. 3 shows the connection of the electrometer amplifier for the charge measurement.

High voltage supply:

- Connect the high voltage cable to the positive pole of the high voltage power supply and the negative pole to earth.
- Mount the insulated stand rod in a saddle base, and put the free point of the high voltage cable (**a**) through the uppermost hole of the insulated stand rod.
- Mount the insulator rod holding the conducting spheres in a saddle base, and put the small conducting sphere on it.

Setup for the charge measurement:

- Supply the electrometer amplifier with voltage from the plug-in unit.
- Attach the Faraday's cup (**b**) with the clamping plug.
- Attach the capacitor 10 nF (**c**).
- Use a connection lead to connect the connection rod (**d**) to earth and, if possible, the earth to the earth of the high voltage power supply through a long connection lead.
- Connect the voltmeter to the output.

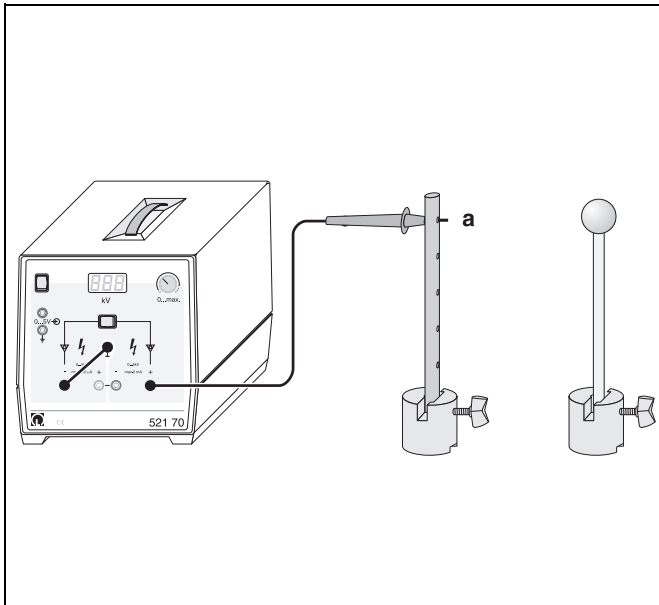


Fig. 2 Setup for charging a sphere in free space.

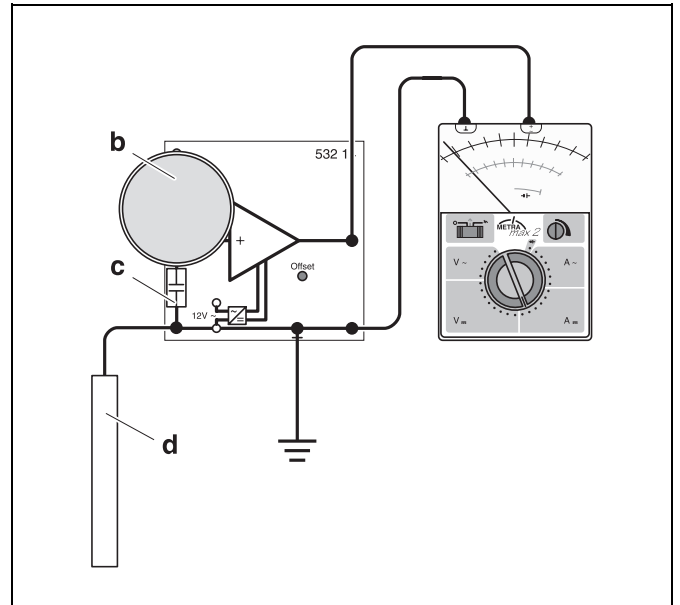


Fig. 3 Connection of the electrometer amplifier for the measurement of the charges.

Carrying out the experiment

- Discharge the conducting sphere by touching it with the connection rod.
- Switch on the high voltage power supply, and set the high voltage U to 1 kV.
- Charge the conducting sphere by touching it with the point of the high voltage cable.

To measure the charge, discharge the Faraday's cup by touching it with the connection rod (d), then take the connection rod in your hand, and move the conducting sphere with the insulator rod to the inside wall of the Faraday's cup (see Fig. 4).

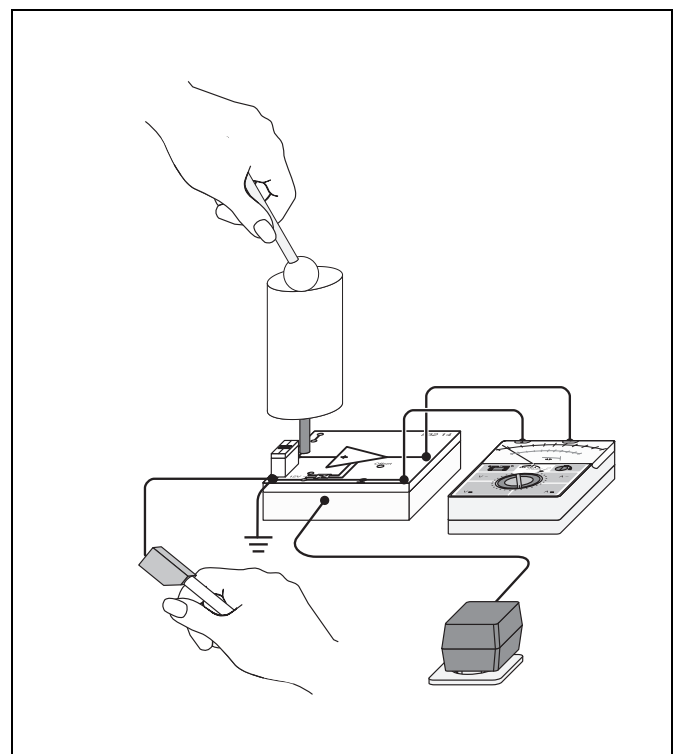
- Increase the high voltage U in steps of 1 kV up to 10 kV, and repeat the measurement each time.
- Perform the same series of measurements with the medium and the large conducting sphere.

Measuring example

Table 1: The charge Q on conducting spheres with different diameters d in as a function of the high voltage U

$\frac{U}{\text{kV}}$	$\frac{Q}{\text{nAs}}$	$\frac{Q}{\text{nAs}}$	$\frac{Q}{\text{nAs}}$
	$d = 2.5 \text{ cm}$	$d = 3.0 \text{ cm}$	$d = 3.5 \text{ cm}$
1.0	1.0	1.3	1.9
2.0	2.5	3.2	3.8
3.0	3.7	4.6	5.4
4.0	4.7	6.1	7.3
5.0	5.7	7.5	9.1
6.0	7.2	9.2	10.8
7.0	8.7	10.9	12.5
8.0	9.4	12.1	14.3
9.0	10.7	13.7	16.2
10.0	11.9	15.3	17.4

Fig. 4 Measurement of the charge on a conducting sphere.



Evaluation and results

In Fig. 5, the charge Q is plotted as a function of the high voltage U for the three conducting spheres. The proportionality is confirmed since, within the accuracy of measurement, the measuring values lie on the straight lines through the origin drawn in the graph.

The slopes of the straight lines through the origin, that is, according to Eq. (I), the capacitances C , are compiled in Table 2 and plotted in Fig. 6 as a function of the diameter d of the sphere.

Tab. 2: The capacitances C of the conducting spheres (determined from Fig. 4) and the corresponding diameters d

$\frac{d}{\text{cm}}$	$\frac{C}{\text{pF}}$
2.5	1.19
3.0	1.53
3.5	1.78

The straight line through the origin drawn in Fig. 6 has the slope

$$a = 50 \cdot 10^{-12} \frac{\text{F}}{\text{m}}$$

Because of Eq. (II), we have . Thus we find for the permittivity of free space

$$\epsilon_0 = 8.0 \cdot 10^{-12} \frac{\text{As}}{\text{Vm}}$$

Value quoted in the literature: $\epsilon_0 = 8.85 \cdot 10^{-12} \frac{\text{As}}{\text{Vm}}$.

The value found in the experiment is smaller than the value quoted in the literature. The reason for this is that the insulator rod reaches into the conducting spheres. The capacitances of the conducting spheres, therefore, turn out to be smaller than those of ideal spheres.

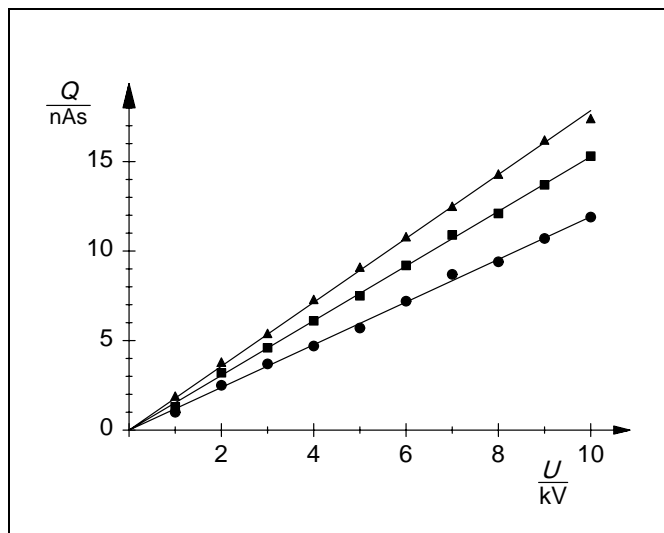


Fig. 5 Dependence of the charge Q on the high voltage U for conducting spheres of different diameters d .
circles: 2.5 cm, boxes: 3.0 cm, triangles: 3.5 cm

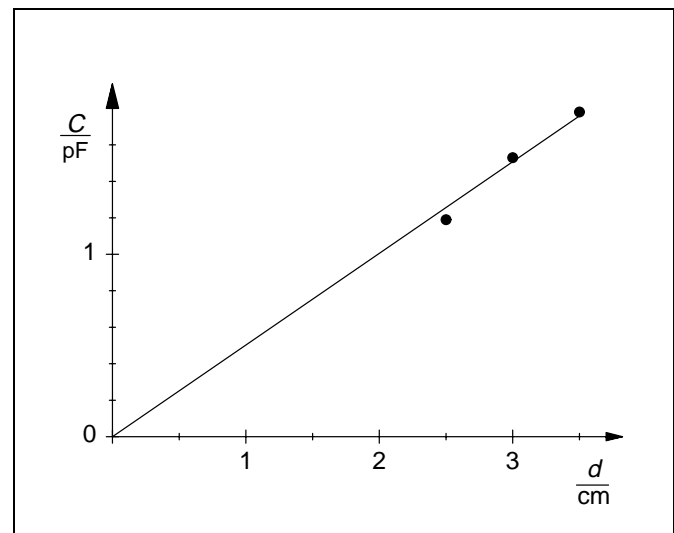


Fig. 6 The capacitance C as a function of the diameter d of the conducting sphere