Investigating the maximum density of water

Objects of the experiments

- Measuring the thermal expansion of water in the temperature range between 0 °C and 15 °C.
- Demonstrating the thermal anomaly and determining the maximum density.

Principles

Water has an important anomaly which shows up when it is warmed up starting from 0 °C: Up to a temperature of about 4 °C, water has a negative expansion coefficient; that is, it shrinks when it is warmed. After passing zero at 4 °C, the expansion coefficient becomes positive. Since the density corresponds to the reciprocal of the volume, this means that water has a maximum density at 4 °C.

In the experiment, the maximum density of water is demonstrated by means of a vessel with a capillary tube. Starting from room temperature, the entire arrangement is cooled down to about 1 °C in an ice-water bath, the water in the vessel being permanently stirred. Alternatively, the arrangement can be slowly warmed up by the vicinity after cooling down in a refrigerator. The water level h in the capillary tube is measured as a function of the water temperature ϑ. The total volume of the water can then be calculated from

$$V(\vartheta) = V_0 + \pi \cdot \frac{d^2}{4} \cdot h(\vartheta) \qquad \text{(I)}.$$

d = 1.7 mm: inner diameter of the capillary tube
V_0 = 310 cm^3: volume of the water in the vessel

Therefore, the density fulfils the equation

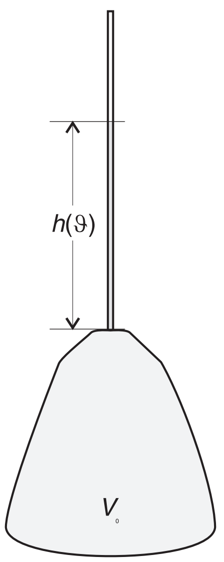
$$\frac{\rho(\vartheta)}{\rho(0°C)} = \frac{V_0 + \pi \cdot \frac{d^2}{4} \cdot h(0°C)}{V_0 + \pi \cdot \frac{d^2}{4} \cdot h(\vartheta)} \qquad \text{(II)}$$

if the fact that the glass vessel (Duran) also expands is neglected. If this expansion is taken into account, Eq. (II) changes into

$$\frac{\rho(\vartheta)}{\rho(0°C)} = \frac{V_0 + \pi \cdot \frac{d^2}{4} \cdot h(0°C)}{V_0 \cdot (1 + 3 \cdot \alpha \cdot \vartheta) + \pi \cdot \frac{d^2}{4} \cdot h(\vartheta)} \qquad \text{(III)}.$$

$\alpha = 3.25\,10^{-6}$ K^{-1}: linear expansion coefficient of Duran

The thermal expansion of the capillary tube is still neglected in Eq. (III).



Determination of the thermal expansion of water in a vessel with a capillary tube from the change of the water level in the capillary tube.

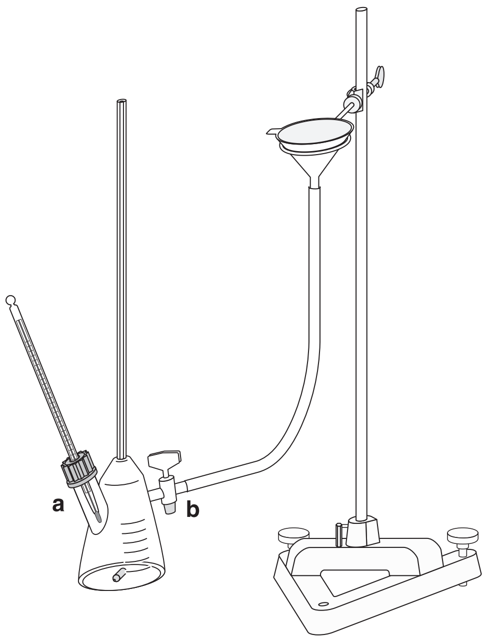
Apparatus

1 device for demonstrating the anomaly of water	667 505
1 magnetic stirrer, without hotplate	666 845
1 thermometer, –10 °C to +40 °C	382 36
or	
1 digital thermometer	666 190
1 temperature sensor, NiCr-Ni	666 193
1 glass tank 300 × 200 × 150 mm^3	664 195
1 funnel	665 008
1 rubber tubing, 12/8 mm dia.	307 66
1 clamp with ring	301 10
1 universal clamp, 0 … 80 mm dia.	666 555
1 Leybold multiclamp	301 01
1 stand rod, 47 cm	300 42
1 stand base, V-shape, 20 cm	300 02

additionally required:

distilled water

refrigerator
or
500 g chopped ice, 50 g table salt



Setup

First:

– Insert the stirring rods via the threaded tube **(a)** into the device for demonstrating the anomaly of water.
– Slide the screw-type fastener with 8-mm fitting over the thermometer

or

– Slide the screw-type fastener with 1.5-mm fitting over the temperature sensor NiCr-Ni.
– Tighten the screw-type fastener at the threaded tube.

Filling in distilled water:

– Connect the funnel to the filling tube **(b)**.
– Pour distilled water into the funnel, open the glass tap, and add water continuously so that the funnel is full at all times while the device is being filled.
– Remove air bubbles by swirling the device.

When the water has reached the upper edge of the capillary tube:

– Close the tap.
– After pouring the remaining water out of the funnel and the rubber tubing, remove the tubing from the filling tube.

Method 1: cooling the water for a measurement at rising temperature

– Set the thermostat of the refrigerator such that the temperature in the bottle compartment is about 0.5–1 °C (check by trying, the water must not freeze).
– Fill distilled water into the device for demonstrating the anomaly of water, and allow the device to cool in the bottle compartment of the refrigerator overnight.

Method 2: preparing a freezing mixture for a measurement at falling temperature

– Prepare a freezing mixture from 450 g of ice and 40 g of table salt in the glass tank and mix thoroughly.

Fig. 1 Filling in distilled water
Here shown as an example:
temperature measurement with a thermometer

Carrying out the experiment

Method 1: measurement at rising temperature

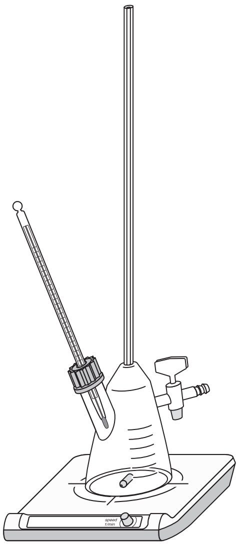


Fig. 2 Experimental setup for the measurement with rising temperature
Here shown as an example:
temperature measurement with a thermometer

- Place the device for demonstrating the anomaly of water on the magnetic stirrer.
- Immediately turn the magnetic stirrer on and set to moderate speed.
- If the temperature sensor NiCr-Ni is used, switch the digital thermometer on and connect the sensor.
- Read the level *h* of the water in the capillary tube as a function of the temperature ϑ, and record it.

Method 2: Measurement at falling temperature

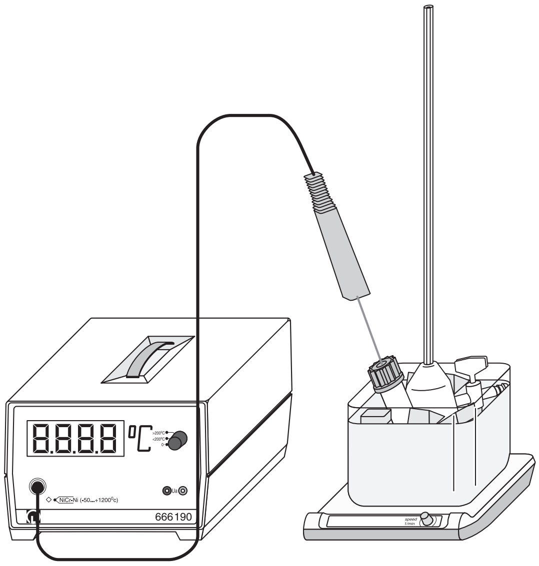


Fig. 3 Experimental setup for the measurement with falling temperature
Here shown as an example:
temperature measurement with the temperature sensor NiCr-Ni

- Put the device for demonstrating the anomaly of water into the glass tank with the freezing mixture and immediately place the tank on the magnetic stirrer.
- Immediately turn the magnetic stirrer on and set to moderate speed.
- If the temperature sensor NiCr-Ni is used, switch the digital thermometer on and connect the sensor.
- Read the level *h* of the water in the capillary tube as a function of the temperature ϑ and record it.

As soon as the temperature falls below 0.5 °C:

- Take the device for demonstrating the anomaly of water out of the freezing mixture (the water must not freeze).

Measuring example

Table 1: The level *h* of the water in the capillary tube as a function of the temperature ϑ, recorded at rising temperature

ϑ	$\frac{h}{\text{mm}}$	ϑ	$\frac{h}{\text{mm}}$	ϑ	$\frac{h}{\text{mm}}$
0.6 °C	24.6	4.0 °C	23.1	7.4 °C	24.6
0.8 °C	24.5	4.2 °C	23.2	7.6 °C	24.7
1.0 °C	24.4	4.4 °C	23.2	7.8 °C	24.8
1.2 °C	24.2	4.6 °C	23.2	8.0 °C	25.0
1.4 °C	24.0	4.8 °C	23.3	8.5 °C	25.6
1.6 °C	23.9	5.0 °C	23.4	9.0 °C	26.0
1.8 °C	23.8	5.2 °C	23.4	9.5 °C	26.6
2.0 °C	23.7	5.4 °C	23.5	10.0 °C	27.0
2.2 °C	23.6	5.6 °C	23.5	10.5 °C	28.2
2.4 °C	23.5	5.8 °C	23.6	11.0 °C	28.9
2.6 °C	23.4	6.0 °C	23.8	11.5 °C	29.8
2.8 °C	23.3	6.2 °C	23.8	12.0 °C	30.6
3.0 °C	23.2	6.4 °C	23.9	12.5 °C	31.5
3.2 °C	23.2	6.6 °C	24.0	13.0 °C	32.7
3.4 °C	23.1	6.8 °C	24.1	13.5 °C	33.2
3.6 °C	23.1	7.0 °C	24.3	14.0 °C	34.7
3.8 °C	23.1	7.2 °C	24.5		

Evaluation

Fig. 4 is a plot of the measuring values of Table 1. By extrapolation, the level *h* of the water in the capillary tube at 0 °C is found to be 25.07 cm. Now the relative density can be calculated according to Eq. (III).

In Fig. 5, the dependence of the relative density on the temperature is shown. The maximum value of 1.00013 is found at ϑ = 3.6 °C.

Values quoted in the literature:

Maximum of the density of water:

$$\rho(3.89\ °\text{C}) = 0.999973\ \text{g cm}^{-3}$$

$$\frac{\rho(3.89°\text{C})}{\rho(0°\text{C})} = 1.000105$$

Results

The volume of water decreases when the temperature increases between 0 °C and about 4 °C and expands only at higher temperatures.

The density of water has its maximum at about 4 °C.

Fig. 4 The level *h* of the water in the capillary tube is a measure for the thermal expansion of water and is shown here as a function of the temperature

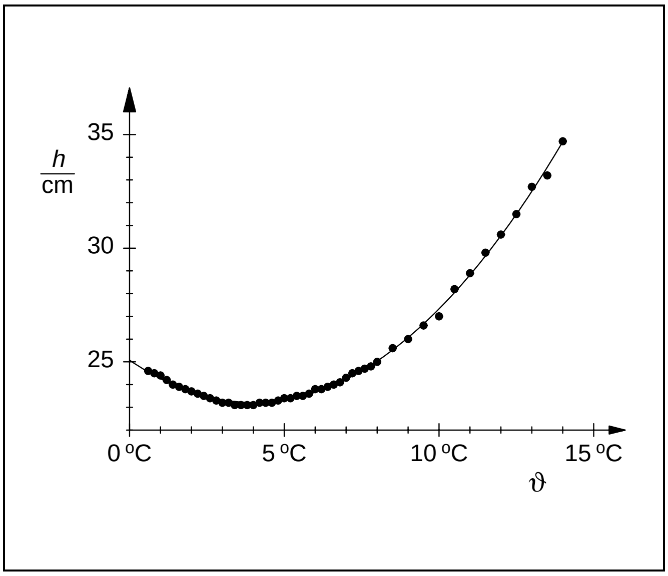


Fig. 5 Relative density of water as a function of the temperature
○: calculated according to Eq. (II),
●: calculated according to Eq. (III).