Verifying the Bernoulli equation – Measuring with a pressure sensor and CASSY

Objects of the experiment

- To verify that the dynamic pressure increases in reduced cross-sectional areas.
- To verify that the flow velocity increases in reduced cross-sectional areas.
- To verify that the volume flow and total pressure remain constant.

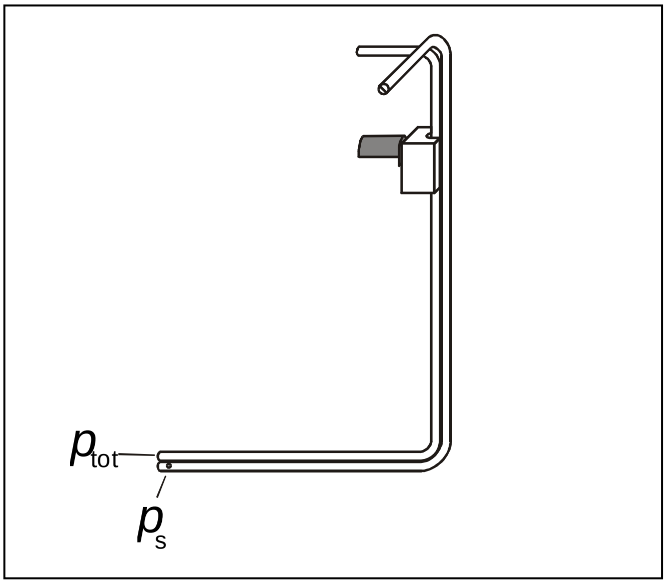
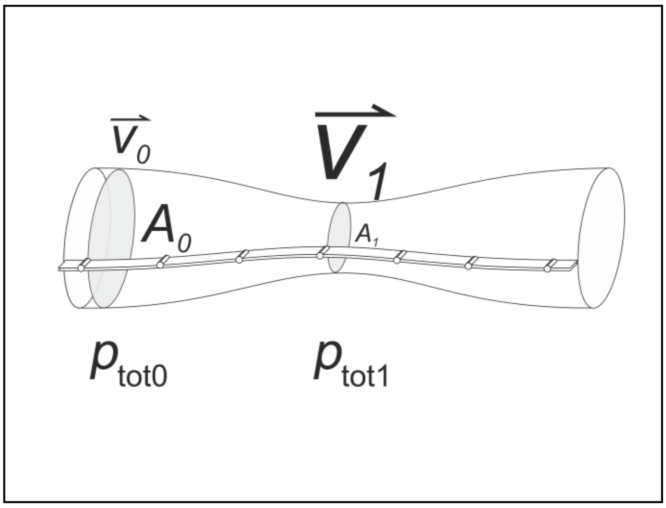


Fig. 1: Prandtl pressure probe for measuring the static pressure p_s and the total pressure p_{tot}.

Fig. 2 Bernoulli's principle: cross-sectional areas A_0 and A_1, flow velocities v_0 and v_1, total pressures p_{tot0} and p_{tot1}. The font size indicates the absolute value of the physical quantity.



Principles

Bernoulli's law states the relationship between static pressure p_s and flow velocity v. The following equation applies to a friction-free, horizontally flowing stream through a stationary flow tube between two points labeled with indices 0 and 1:

$$p_{s0} + \frac{\rho}{2}v_0^2 = p_{s1} + \frac{\rho}{2}v_1^2 \qquad \text{(I)}$$

Density of the air: $\rho = 1.2\,\frac{\text{kg}}{\text{m}^3}$

In particular, equation (I) states that the total pressure p_{tot} has the same value everywhere:

$$p_{tot} = p_s + p_d = \text{const.} \qquad \text{(II)}$$

p_d: dynamic pressure

In the experiment described here air flows through a narrowing wind tunnel. Its cross-sectional area is reducing from 0.020m² to 0.015m². We will measure the total pressure p_{tot} and the static pressure p_s at several positions in the wind tunnel.

The flow velocities v_0 and v_1 at two different locations in the wind tunnel with cross-sectional areas A_0 and A_1 are given by the continuity equation:

$$v_0 \cdot A_0 = v_1 \cdot A_1 \qquad \text{(III)}$$

The continuity equation states that the volume flow $J = v \cdot A$ in the tube is constant. The incompressibility of air can be assumed for the occurring flow velocities in this experiment.

The definition of the dynamic pressure $p_d = \frac{\rho}{2}v^2$ can be derived from equations (I) and (II). It allows to eliminate v_0 in equation (III). By rearranging we obtain:

$$v_1 = \sqrt{\frac{2}{\rho}p_{d0}} \cdot \frac{A_0}{A_1} \qquad \text{(IV)}$$

with

$$p_d = p_{tot} - p_s \qquad \text{(V)}$$

The dynamic pressure p_d is determined by measuring the pressure difference with the Prandtl pressure probe. The cross-sectional area sizes are written on the ramp of the wind tunnel.

GENZ 2015-06

Apparatus

1 Wind tunnel 373 12
1 Prandtl pressure probe 373 13
1 Suction and pressure fan 373 041
1 Measurement trolley for wind tunnel 373 075

1 Sensor-CASSY 2 .... 524 013
or
1 Mobile-CASSY 524 009A
or
1 Pocket-CASSY 2 Bluetooth 524 018

1 CASSY Lab 2 .... 524 220

1 Pressure sensor S, ±70 hPa 524 066

Additionally required: 1 PC with Windows XP or higher

Safety notes

Mind the safety notes in the instruction sheets of the wind tunnel and the suction and pressure fan.

Before removing the protective grid or the nozzle:

- Pull out the mains plug and
- wait for at least 30 seconds until the suction and pressure fan comes to a complete stop.

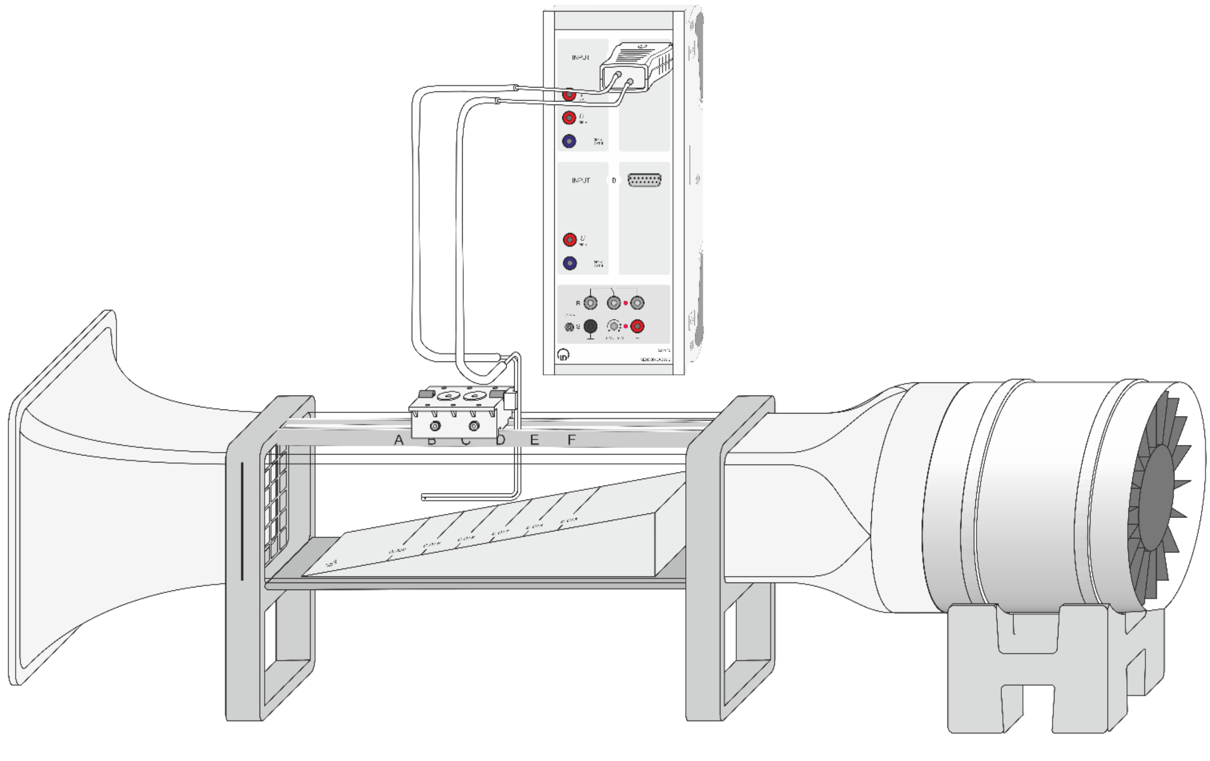
Setup

Assemble the wind tunnel and the fan as shown in Fig 3. Plug the suction side of the fan into the outlet nozzle so that the air is drawn through the wind tunnel during the experiment. Ensure a clearance of approx. 1 m in front of the suction nozzle and behind the fan so that the air can be drawn into the wind tunnel without any turbulence.

– Insert the lamination filter (protection grid) into the slot at the entry gate.
– With the aid of the four screws fix the Bernoulli ramp underneath the Plexiglas hood so that the ramp height increases in the direction of flow.
– Mount the scaled sealing strip (included in the equipment for the wind tunnel) on the guiding rails.
– Carefully push the Prandtl pressure probe through the foam rubber seal of the sealing strip. Push the Prandtl pressure probe down so that it reaches a position approx. 2 cm higher than the highest point of the ramp.
– Plug the Prandtl pressure probe onto the measurement trolley.
– Slide the front of the measurement trolley to Position A of the sealing strip.
– Plug the Pressure sensor S, ±70 hPa into Input A of Sensor-CASSY 2 (Fig. 3) or into the Mobile-CASSY (Fig. 5).
– Connect the 3 mm hose coming out of pressure connection p_1 (top) of pressure sensor S to the 5 mm hose delivered with the Prandtl pressure probe.
– Connect the other end of the 5 mm hose to the Prandtl pressure probe outlet for p_{tot} (see Fig. 1).
– In the same way, connect pressure connection p_2 (bottom) of the pressure sensor S to the p_s outlet of the Prandtl pressure probe (see Fig. 1).

Remark: For further hints refer to instruction sheets 373 12, 373 13, 373 041 and 524 066.

Fig. 3: Experimental setup with Sensor-CASSY 2.

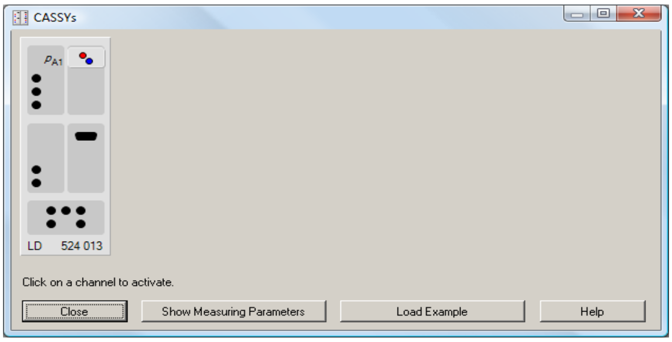


Carrying out the experiment

- If not yet installed, install the software CASSY Lab 2 and open the software.

a) Measuring with Sensor-CASSY 2

- Load the settings in CASSY Lab 2.
- The connected pressure sensor S should be displayed if Sensor-CASSY 2 is powered on and connected to the computer via the USB port.
- Activate the connected pressure sensor S at Input A by clicking on the pressure sensor S.



Remark: Further details about connecting sensors to Sensor-CASSY 2 can be found in the CASSY Lab 2 manual or in the web help.

- Reset the pressure sensor S by clicking →0← in the "*Settings*" pane when "*relative pressure* p_{A1}" is marked.
- Set the suction and pressure fan to its minimum speed (i.e. left limit position of fan control) and only then switch it on.
- Slowly increase the speed of the suction and pressure fan until the "*relative pressure* p_{A1}" reaches approx. 8 Pa at position A.
- To record the pressure values with CASSY Lab 2 click when table *"p_d(A) [autom.]"* is displayed. (In this experiment: *"relative pressure* p_{A1}" is identical to p_d).
- Place the measurement trolley with the Prandtl pressure probe one position further.
- Repeat this measurement for the positions "B" to "F".
- After six single measurements start a new series of measurements by clicking #1. Increase the fan speed a bit and put the measurement trolley back to position A.
- Repeat these steps for two more fan speeds.

Remark: For the reset button →0← to appear in the "Settings" pane "relative pressure p_{A1}*" has to be marked in the submenu of "CASSYs". It is recommended to press the reset button →0← before each series of measurements.*

b) Measuring with Mobile-CASSY

*Remark: To record the pressure values with Mobile-CASSY automatically, follow the instructions as described in **a)**.*

- Switch on the Mobile-CASSY with the MENU key.
- Open the main menu by pressing the MENU key again.
- Select the submenu "Quantities" by using the arrow keys, or , and enter the submenu using the right key.
- Go to the submenu "p" using the right key.
- Choose "Compensate Offset" and set the pressure to zero by pushing the right key.
- Press MENU key and then the left key to display the current pressure value.
- Set the suction and pressure fan to its minimum speed (i.e. left limit position of fan control) and only then switch it on.

Remark: For further hints on using Mobile-CASSY (524 009A) refer to the instruction sheet.

- Slowly increase speed of the suction and pressure fan until the pressure difference Δp (= p_d) reaches approx. 8 Pa at position A.
- Read off the dynamic pressure p_d.
- Load the settings in CASSY Lab 2 and type in the pressure value in table "*p_d(A) [manu.]*".
- Enter the corresponding cross-sectional area (e.g.: 0.020 m^2) in column A of table "*p_d(A) [manu.]*" (left side of CASSY Lab 2 window).
- Place the measurement trolley with the Prandtl pressure probe one position further.
- Repeat this measurement for the positions "B" to "F".
- After six single measurements start a new series of measurements by clicking #1. Increase the fan speed a bit and put the measurement trolley back to position A.
- Repeat these steps for two more fan speeds.

Remark: To record more than the three prepared series of measurements open "Measurement" in the menu bar and select "Append new Measurement Series". Then click when table "p_d(A) [manu.]" is displayed. Open the "Settings" pane and mark "p_d(A) [manu.]" in the submenu "Displays". Push the button "Add new Curve" and select "p_d#4" in the drop down menu for "y-axis".

Measuring example

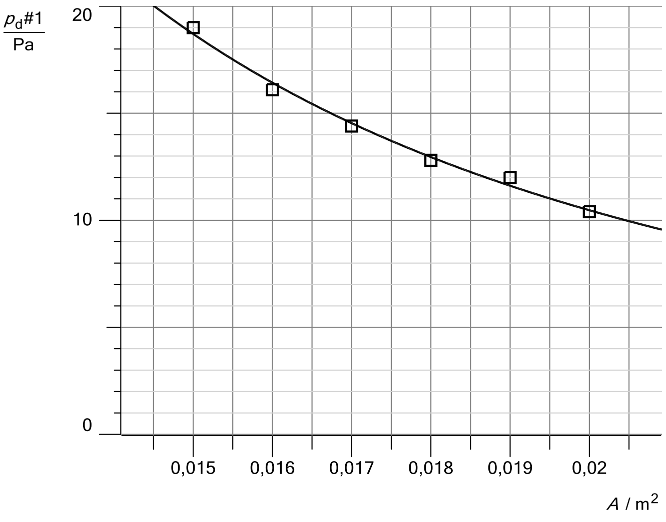


Fig. 4: Dynamic pressure p_d as function of the cross-sectional area A. The solid curve corresponds to a fit of a $1/A^2$ hyperbola.

Tab. 1: Dynamic pressure p_d at positions A to F for a constant fan speed.

Position	$\frac{A}{m^2}$	$\frac{p_d}{Pa}$
A	0.020	10.4
B	0.019	12.0
C	0.018	12.8
D	0.017	14.4
E	0.016	16.1
F	0.015	19.0

Results

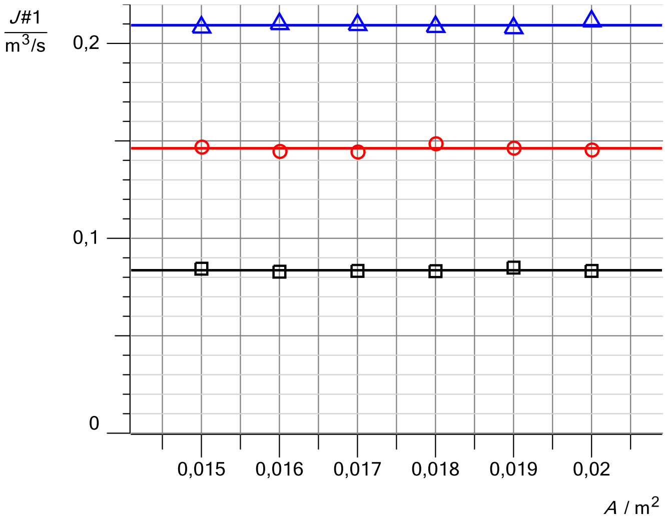
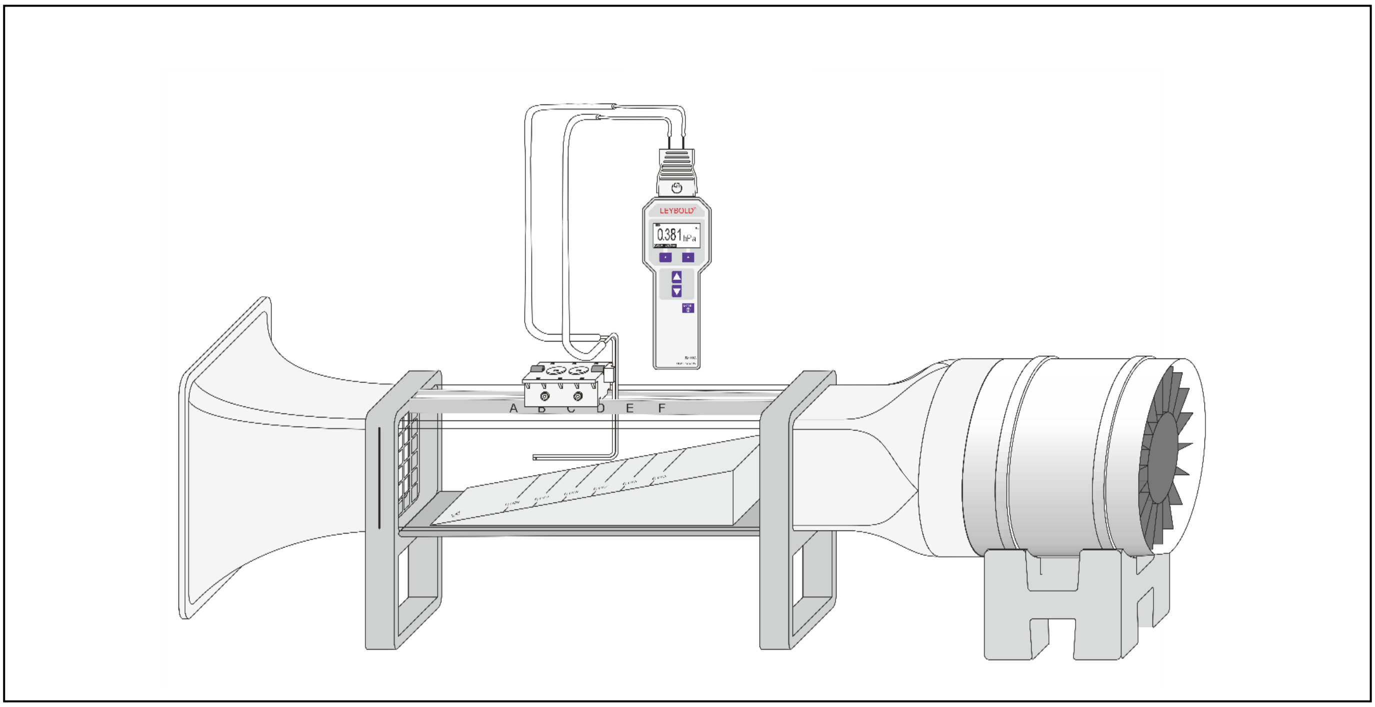


Fig. 6: Volume flow J as a function of the cross-sectional area A for three different wind speeds of the fan. The solid lines indicate the average value of one series of measurement.

Tab. 2: Flow velocity v and volume flow J calculated with the values of Tab. 1.

Position	$\frac{v}{\frac{m}{s}}$	$\frac{J}{\frac{m^3}{s}}$
A	4.2	0.083
B	4.5	0.085
C	4.6	0.083
D	4.9	0.083
E	5.2	0.083
F	5.6	0.084

Fig. 5: Experimental setup with Mobile-CASSY.



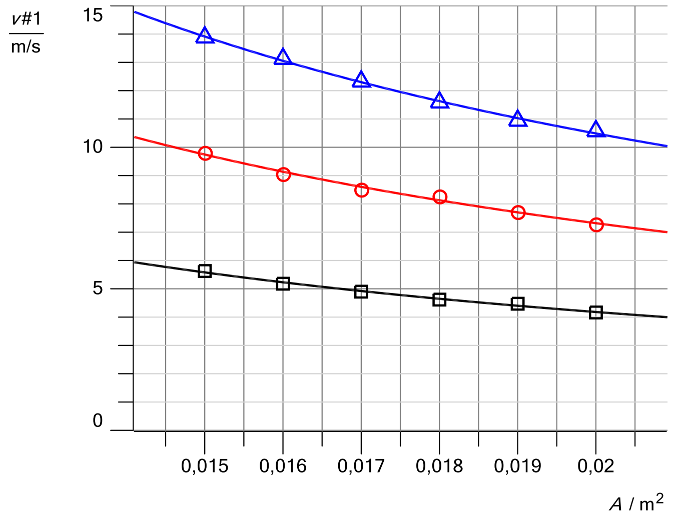


Fig. 7: Flow velocity v as a function of cross-sectional area A for three different wind speeds of the fan. The solid curve corresponds to a fit of a $1/A$ hyperbola.

Evaluation

The relation

$$p_d \propto \frac{1}{A^2}$$

derived from Bernoulli's principle and the continuity equation seems to be confirmed by Tab. 1 and Fig. 4. The best fit function is a quadratic hyperbola.

Equation (IV) seems to be confirmed by Tab. 2 and Fig. 7:

$$v \propto \frac{1}{A}$$

(Density of air: $\rho = 1.2\ \frac{\text{kg}}{\text{m}^3}$)

The flow velocity increases with decreasing cross-sectional area A. The best fit function is a hyperbola.

The volume flow

$J = v\,A$

is constant for all cross-sectional areas (Fig. 6). The minor changes can be explained trough measuring tolerance and leakages. Thus the predictions based on Bernoulli's principle and the continuity equation are verified qualitatively.

Supplementary information

Additionally, the stability of the total pressure p_{tot} along the decreasing cross-sectional area A of the wind tunnel can be verified directly. Therefore only one hose is connected to the total pressure outlet (Fig. 1) of the Prandtl pressure probe as shown in Fig. 8. The hose leads to pressure connection p_1 (top) of pressure sensor S.

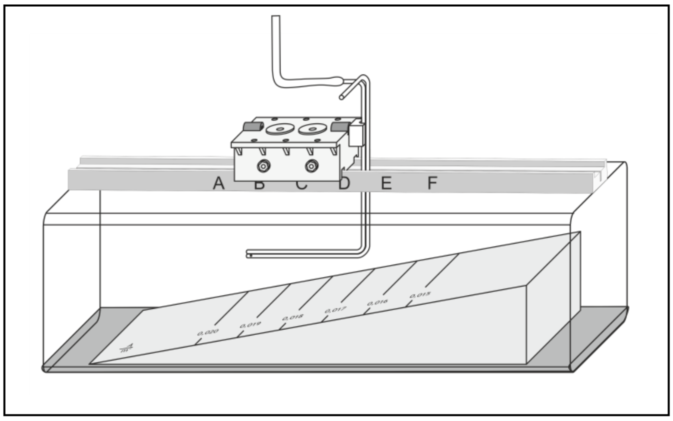


Fig. 8: Experimental setup for measuring the total pressure p_{tot}.

The decreasing static pressure p_s in reducing cross-sectional areas can be verified, too. Therefore only one hose is connected to the static pressure outlet (Fig. 1) of the Prandtl pressure probe (not shown in Fig. 8). The hose leads to pressure connection p_1 (top) of pressure sensor S.

Remark: The decreasing static pressure p_s in reduced cross-sectional areas has already been verified in P1.8.5.4.