Mechanics

Oscillations

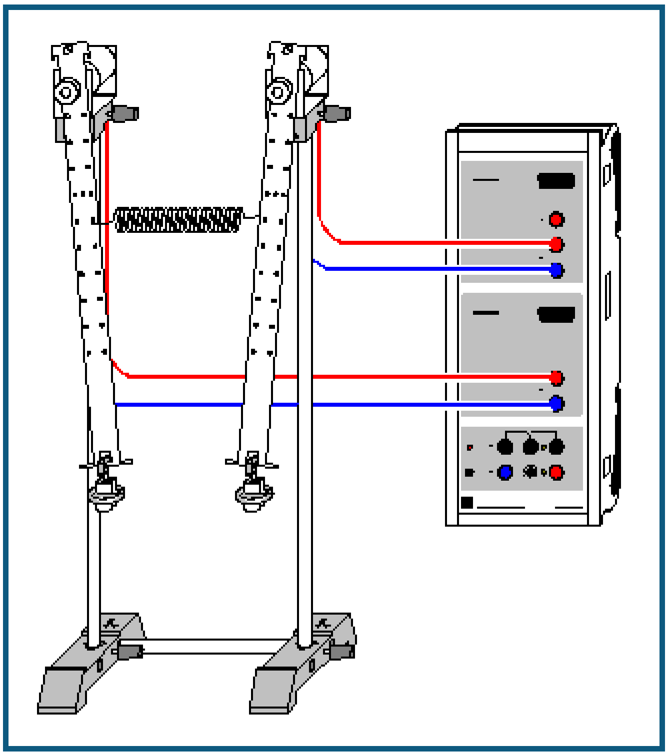
Coupling of oscillations

Coupled pendulum - Recording and evaluating with CASSY

Description from CASSY Lab 2

For loading examples and settings, please use the CASSY Lab 2 help.

Coupled pendulums with two tachogenerators



can also be carried out with Pocket-CASSY

Experiment description

Two coupled pendulums swing in phase with a frequency f_1 when they are deflected from the rest position by the same distance. When the second pendulum is deflected in the opposite direction, the two pendulums oscillate in opposing phase with the frequency f_2. Deflecting only one pendulum generates a coupled oscillation with the frequency

$f_n = ½ (f_1 + f_2)$

in which oscillation energy is transferred back and forth between the two pendulums. The first pendulum comes to rest after a certain time, while the second pendulum simultaneously reaches its greatest amplitude. The time from one standstill of a pendulum to the next is called T_s. For the corresponding beat frequency, we can say

$f_s = | f_1 – f_2 |$.

Equipment list

1	Sensor-CASSY	524 010 or 524 013
1	CASSY Lab 2	524 220
1	Pair of bar pendulums, 40 cm	346 03
1	Set of weights	340 85
2	Support clips, for plugging in	from 314 04ET5
1	Helical spring, 3 N/m	352 10
2	STE tachogenerators	579 43
2	Clamping blocks MF	301 25
1	Stand rod, 25 cm, d = 10 mm	301 26
2	Stand rods, 50 cm, d = 10 cm	301 27
2	Stand bases MF	301 21
2	Pairs of cables, 100 cm, red and blue	501 46
1	PC with Windows XP/Vista/7/8	

Experiment setup (see drawing)

The motions of the pendulums are transmitted to the tachogenerators. The voltages of the tachogenerators are measured at Sensor-CASSY inputs A and B. To vary the coupling, the coupling spring can be connected at different heights.

Carrying out the experiment

- Load settings
- Start the measurement with and deflect both pendulums in phase (measurement stops after 30 s).
- Start the measurement with and deflect both pendulums in opposing phase (measurement stops after 30 s).
- Start the measurement with and deflect only the first pendulum (measurement stops after 30 s).

Evaluation

Click on the **Natural Oscillations** display to view the two natural oscillations:

$U_+ = U_A + U_B$

$U_- = U_A – U_B$

When the pendulums are excited in phase, only U_+ oscillates with a frequency f_1; only U_- oscillates for opposing phase oscillation and has the frequency f_2. Only when the single pendulum alone is deflected does the system oscillate with both natural frequencies, thus generating the typical beat in the **Standard** display.

To determine the beat frequency f_s and the new oscillation frequency f_n, you can mark e.g. the diagram with vertical lines or measure the difference directly (to increase the accuracy you should average the values over several periods when determining the oscillation frequency f_n).

In this example we obtain $f_1 = 0.875$ Hz, $f_2 = 0.986$ Hz, $f_n = 0.93$ Hz, $f_s = 0.11$ Hz, which closely confirms the theory $f_n = ½ (f_1 + f_2) = 0.93$ Hz and $f_s = | f_1 – f_2 | = 0.11$ Hz.

In the **Frequency Spectrum**, you can compare the frequencies and amplitudes U_+, U_- and U_A. The easiest way to determine the frequency is to find the peak centers.

Theory

When we apply suitable approximations (small deflections, negligible weight of coupling spring and pendulum bar, no damping), the motion equations of the pendulum bodies are as follows:

$F_1 = ma_1 = –Dx_1 + C(x_2 – x_1)$
$F_2 = ma_2 = –Dx_2 – C(x_2 – x_1)$

$-Dx_i$ (where D = mg/l) represents the restoring force of the individual pendulum, and $C(x_2 – x_1)$ describes the force of the coupling between the two pendulums. These resolve to the superposition

$x(t) = A \cos(\omega_1 \cdot t) + B \cos(\omega_2 \cdot t)$

with the fundamental frequencies ω_1 and ω_2. The specific initial conditions provide the values for A and B:

In-phase excitation gives us $A = x_0$, $B = 0$ (harmonic oscillation with ω_1)
Opposing phase excitation gives us $A = 0$, $B = x_0$ (harmonic oscillation with ω_2)
Deflection of one pendulum gives us $A = B = ½ x_0$.

In the latter case we can say:

$x(t) = ½ x_0 (\cos(\omega_1 \cdot t) + \cos(\omega_2 \cdot t)) = x_0 \cos(½\omega_s \cdot t) \cos(\omega_n \cdot t)$

where $\omega_s = | \omega_1 – \omega_2 |$ and $\omega_n = ½ (\omega_1 + \omega_2)$ or $f_s = | f_1 – f_2 |$ and $f_n = ½ (f_1 + f_2)$.

When the difference between frequencies f_1 and f_2 is small, this equation describes an oscillation with the frequency f_n which is modulated by the slower frequency f_s – i.e. a beat.