Free rotational oscillations

Measuring with a hand-held stop-clock

Objects of the experiment

- Measuring the amplitude of rotational oscillations as function of time
- Determination of the damping constant and the logarithmic decrement
- Investigating the transition form the weekly damped oscillation case to the limit case

Principles

Oscillations (and wave) phenomena are well known due to their presence everywhere in nature and technique. Their investigation is thus both from experimental point of view as from theoretical point of view an important topic as it allows to study fundamental methods and concepts of physics.

The rotary oscillations are a special case among various mechanical oscillator models (compound pendulum, spring pendulum etc.) which allow to investigate the most important phenomena which occur in all types of oscillations. Additionally, to the usual observation of a free damped harmonic oscillator anharmonic oscillator can also be realized. In experiment P1.5.3.4 anharmonic chaotic rotary oscillations are examined in order to show that the harmonic oscillations are only a special case.

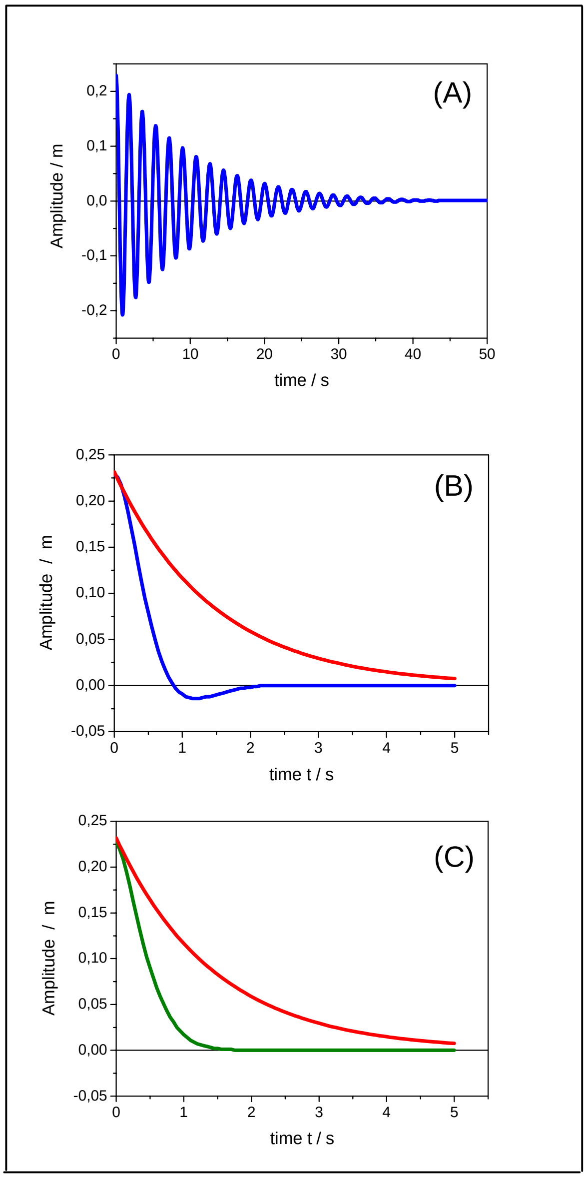

Fig. 1: Schematic representation of various damped oscillation curves:
(A) weakly damped case: $\omega_0^2 > \delta^2$ (blue curve)
(B) heavily damped case: $\omega_0^2 < \delta^2$ (red curve)
in comparison with an damped oscillation of type (A, blue)
(C) aperiodic limit case: $\omega_0^2 = \delta^2$ (green curve)
in comparison with heavily damped case (B, red).

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Apparatus

1 Torsion pendulum 346 00
1 DC power supply 0…16V/0…5 A...................... 521 545
1 Ammeter, DC, I ≤ 2 A, e.g. LD analog 20 .... 531 120
1 Connecting lead, 100 cm, blue 500 442
1 Pair cables, red and blue, 100 cm 501 46
1 Stop clock 313 07

The movement of a free damped (rotary) oscillating system can be described by the differential equation

$$J\frac{d^2\varphi}{dt^2}+k\frac{d\varphi}{dt}+D\cdot\varphi=0 \qquad \text{(I)}$$

J: moment of inertia

D: directional quantity (restoring torque)

k: damping coefficient (coefficient of friction)

φ: angle of rotation

With the damping constant

$$\delta=\frac{k}{2\cdot J} \qquad \text{(II)}$$

the natural angular frequency of an undamped oscillation

$$\omega_0=\sqrt{\frac{D}{J}} \qquad \text{(III)}$$

and the angular frequency of the damped oscillation

$$\omega=\sqrt{\omega_0^2-\delta^2} \qquad \text{(IV)}$$

equation (I) may be resolved by

$$\varphi(t)=\varphi_0\cdot e^{-\delta\cdot t}\cdot\cos\omega\cdot t \qquad \text{(V)}$$

φ_0: initial angel of rotation at time t = 0

δ: damping constant

ω_0: characteristic frequency of an “undamped” system

ω: angular frequency of the damped oscillation

From equation (V) follows that the amplitude decreases by the amplitude factor $e^{-\delta\cdot t}$ (Fig. 1 – case (A)). Thus after a time $1/\delta$ the amplitude has decreased to 1/e of its initial value φ_0. Moreover, from equation (V) follows that the ratio of two successive amplitudes φ_n and φ_{n+1} is constant

$$\frac{\varphi_n}{\varphi_{n+1}}=q=e^{-\delta\cdot T} \qquad \text{(VI)}$$

q: damping ratio

The exponent is called the logarithm decrement

$$\Lambda=\delta\cdot T=\ln\frac{\varphi_n}{\varphi_{n+1}}=\ln q \qquad \text{(VII)}$$

However, according to equation (V) oscillations occur only when the angular frequency (i.e. equation (IV)) has a positive radiant (Fig. 1: case (A)):

$$\omega_0^2>\delta^2$$

If $\omega_0^2<\delta^2$ the solution has the form

$$\varphi(t)=\varphi_0\cdot e^{-\delta\cdot t}\,(e^{\omega\cdot t}+e^{-\omega\cdot t}) \qquad \text{(VIII)}$$

The oscillating system approaches the equilibrium position asymptotically after one oscillation (so called creeping or heavily damped case – Fig. 1 case (B) red curve). The higher the damping constant the slower the approach to zero.

If $\omega_0^2=\delta^2$ the solution has the form

$$\varphi(t)=(\varphi_0+b\cdot t)\,e^{-\delta\cdot t} \qquad \text{(IX)}$$

The damping is so great that there is just no longer a crossing through the rest position. Any reduction in damping leads to an oscillation. This is the so-called aperiodic limit case which is of practical importance because the time required to reach the zero position is minimal. A measuring instrument having a pointer of a moving-coil system is thus designed with aperiodic damping (Fig. 1 case (C) green curve).

In this experiment a rotatable metal wheel with inertia J is used as an oscillator. A helical spring acts on the wheel when its displaced by an angle φ from its rest position to produce a restoring torque M which is approximately given by

$$M=-D\cdot\varphi \qquad \text{(X)}$$

Owing to the unavoidable frictional forces (in the ball baring etc.) the amplitude of mechanical oscillation decreases in time. As a result a free damped oscillation is produced. In many (**but not in all!**) cases, the frictional forces (torques) are proportional to the (angular) velocity in the first order of approximation:

$$M_F=-k\cdot\frac{d\varphi}{dt} \qquad \text{(XI)}$$

On the torsion pendulum the damping according equation (XI) is realized by passing the metal wheel through the field of an electromagnet. The electrons experience the Lorentz force. Thus the electrons are displayed perpendicular to the field of the electromagnet and the direction of the moving wheel. They flow back through the field free part of the wheel (Fig. 2). As a result a closed eddy-current I_{eddy} circuit is produced.

The part of the metal wheel in the magnetic field acts like a moving current carrying conductor on which a force F opposed to the direction of motion and proportional to the velocity v acts. This generates a deaccelerating torque M_F.

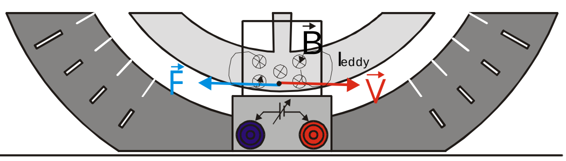


Fig. 2: Generation of eddy currents I_{eddy}.

Setup

Set up the experiment as shown in Fig. 3. The time is measured by the stop clock (not shown in Fig. 3).

Set the pointer of the metal wheel (3a) to the zero position of the scale by turning the drive wheel (3e).

Safety notes

- The current through the eddy current brake should not exceed 2 A for a long time.

Carrying out the experiment

a) Investigating the damping of the oscillation

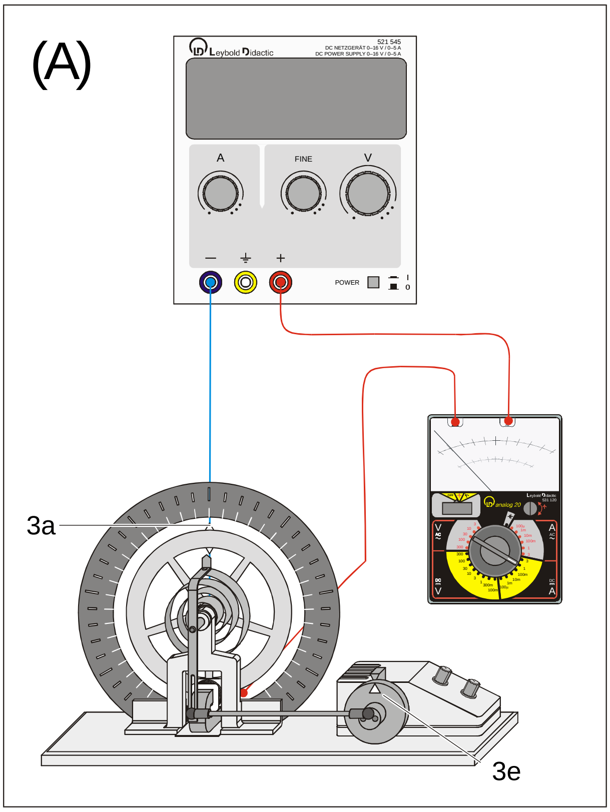
- Set the current for the electromagnet to a small value, e.g. I = 0.18 A
- Move the pointer of the pendulum to the limit position and read off the amplitude A on the same side of the scale after each oscillation T (for the case of weak damping after 5 or 10 oscillations).
- Additionally, measure several times the time for 10 oscillations to determine the oscillation period T.

Hint: If the pendulum achieves an equilibrium in less then 10 oscillations measure the time several times to obtain the mean value.

- Repeat the experiment in the same way for a larger current (i.e. I = 0.4 A).

b) Investigating the transition from oscillation to the limit cases

- Increase the current until the pendulum performs an oscillation depicted by the blue curve in Fig. 1 (B).
- Move the pointer of the pendulum to the limit position and measure the time taken for an oscillation until the equilibrium position is reached. Determine the oscillation period as mean value from several measurements.
- Increase the current until the pendulum performs an oscillation depicted by the green curve in Fig. 1 (C).
- Measure the time taken by the pendulum when released from the limit position. Determine mean value from e.g. 5 measurements.



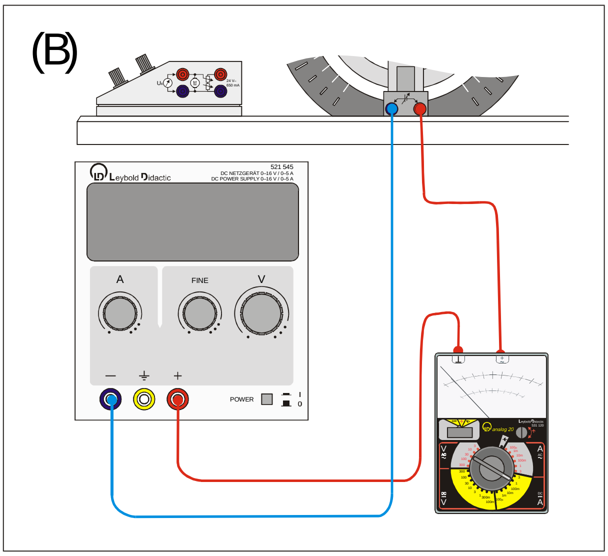


Fig. 3: Experimental setup (wiring diagram schematically) for observing damped rotational oscillations.

Measuring example

a) Investigating the damping of the oscillation

Note: The experimental data may differ from pendulum to pendulum due to inevitable tolerances between the eddy current brakes and the tiny differences in the mechanical set up.

Table. 1: Measured oscillation amplitude A as function of time n·T (n-times the oscillation period) for I = 0.18 A and I = 0.25 A.

$\frac{t}{s}$	$\frac{A}{Scd}$	$\frac{t}{s}$	$\frac{A}{Scd}$
0.0	20.2	0.0	20.2
1.8	18.7	1.8	17.2
3.5	17.3	3.6	14.5
5.3	16.3	5.3	12.3
7.1	15.2	7.2	10.4
8.9	14.2	9.0	8.7
10.7	13.2	10.8	7.5
12.5	12.3	12.6	6.3
14.3	11.4	14.5	5.3
16.2	10.6	16.3	4.5
17.9	9.9	18.2	3.8
19.8	9.1	20.0	3.2
21.6	8.5	21.9	2.7
23.5	7.9	23.8	2.2
25.3	7.3	25.6	2.0
27.1	6.9	27.5	1.7
28.9	6.3	29.3	1.4
30.8	5.9	31.2	1.3
32.7	5.5		
34.1	4.1		
35.9	3.7		
37.7	3.4		
39.5	3.0		
41.3	2.8		
43.1	2.5		
44.9	2.2		
46.7	2.0		
48.5	1.7		
50.3	1.6		
52.1	1.3		
53.9	1.1		
55.7	1.0		
57.5	0.9		
59.2	0.7		
61.0	0.5		

Table. 2: Oscillation period (mean value determined by 5 measurements) for different eddy currents.

Eddy current $\frac{I}{A}$	Oscillation period $\frac{T}{s}$
0.18	1.80
0.25	1.82

b) Investigating the transition from oscillation to the limit cases

I = 1.3 A.

Measured oscillation period: 2.14 s.

I = 1.5 A

Measured oscillation period: 1.9 s

Evaluation and results

a) Investigating the damping of the oscillation

Fig. 4 summarizes the result of Table 1. The damping constant δ can determined for instance by fitting equation (V) to the experimental data. Alternatively, the fit of a straight line to data plotted in Fig. 5 gives the damping constant δ from which the logarithmic decrement Λ can be determined (Table 3.).

Table. 3: Oscillation period T (from Table 2). damping constant δ (determined by a fit to the experimental data plotted in Fig. 5) and logarithmic decrement Λ for various eddy currents I.

$\frac{I}{A}$	$\frac{T}{s}$	$\frac{\delta}{s^{-1}}$	Λ
0.18	1.80	0.039	0.07
0.25	1.82	0.094	0.17

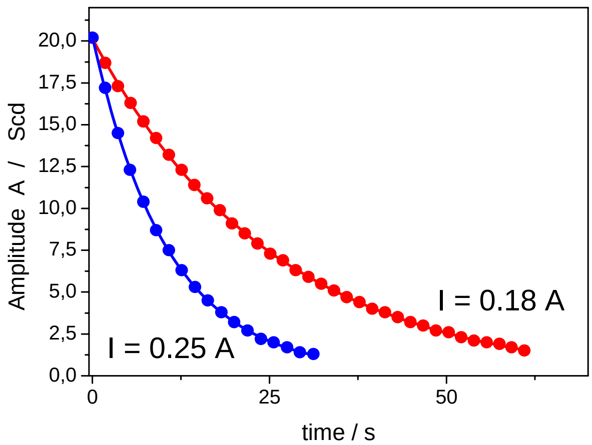


Fig. 4: Amplitude as function of time. The solid lines correspond to a fit according equation (V).

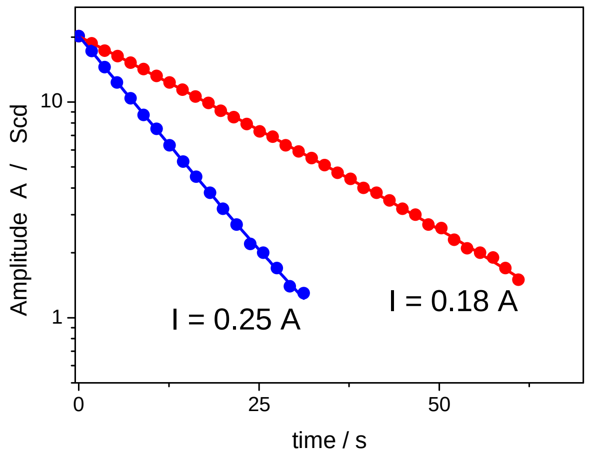


Fig. 5: Amplitude as function of time. The solid lines correspond to a fit of a straight line yielding the damping constants δ listed in Table 3.

b) Investigating the transition from oscillation to the limit cases

The pendulum reaches the equilibrium after one oscillation for

I = 1.3 A. The measured oscillation period is 2.14 s.

For I = 1.5 A the pendulum reaches the equilibrium in 1.9 s without oscillating over the zero position.

In this so-called aperiodic case the adjustment time required by the system to return to the equilibrium is a minimum.

Supplementary information

The oscillations with a restoring torque described by equation (X) are called harmonic oscillations. The harmonic oscillator is only a special case among systems which are capable of oscillation. Most of the real oscillations are not harmonic, i.e. relation (X) is not strictly satisfied. However, many oscillations can be considered as harmonic oscillations at least in the first approximation by developing the restoring torque (forces) as function about the rest position in series and neglecting non-linear terms. The equation of motion (I) of such on oscillating system can generally not be solved analytically.

The anharmonic oscillator is investigated in experiment P1.5.3.4.