

Kinetic energy of a uniformly accelerated mass

Recording and evaluating with VideoCom

Objects of the experiments

- Recording the path-time diagram of uniformly accelerated motions in dependence on the accelerated mass and the accelerating force.
- Determining the kinetic energy and the total energy of the uniformly accelerated mass.

Principles

Kinetic energy:

If a mass point with the mass m is uniformly accelerated by a constant force F along a path s , the energy transferred to this mass point is

$$E = F \cdot s \quad (I).$$

If the mass point is at rest in the beginning, the relation between the path s and the time t required to cover it is

$$s = \frac{1}{2} a \cdot t^2 \quad (II),$$

where

$$a = \frac{F}{m} \quad (III)$$

is the constant acceleration; the final velocity is

$$v = a \cdot t \quad (IV).$$

Thus the mass point has the energy

$$E_{\text{kin}} = \frac{1}{2} \cdot m \cdot v^2 \quad (V),$$

which is called kinetic energy.

Potential energy and total energy:

The accelerating force F can stem, e.g., from a second mass m_2 in the gravitational field of the earth which pulls the first mass via a thread (see Fig. 1). Then the force is

$$F = m_2 \cdot g \quad (VI).$$

At the beginning the mass m_2 is located at a height h and, according to Eq. (I), has the energy

$$E_{\text{pot}} = m_2 \cdot g \cdot h \quad (VII);$$

this energy is called potential energy. While the first mass covers the path s , the second mass falls by the same path length. During the fall its potential energy therefore is

$$E_{\text{pot}} = m_2 \cdot g \cdot (h - s) \quad (VIII).$$

As the falling mass is accelerated, too, it has to be taken into account in calculating the kinetic energy. Therefore the total mass

$$m = m_1 + m_2 \quad (IX)$$

has to be inserted as mass m in Eqs. (III) and (V).

Recording and evaluating with VideoCom:

In the experiment, the kinetic energy of a uniformly accelerated slider on an linear air track is studied. The motions of the slider are recorded with the single-line CDD camera VideoCom, which illuminates a retroreflecting foil attached to the slider with LED flashes and images the reflected flashes on a CCD line with 2048 pixels with a camera lens (CCD: charge-coupled device). Up to 80 times per second the present position of the slider is transferred to a computer via a serial interface.

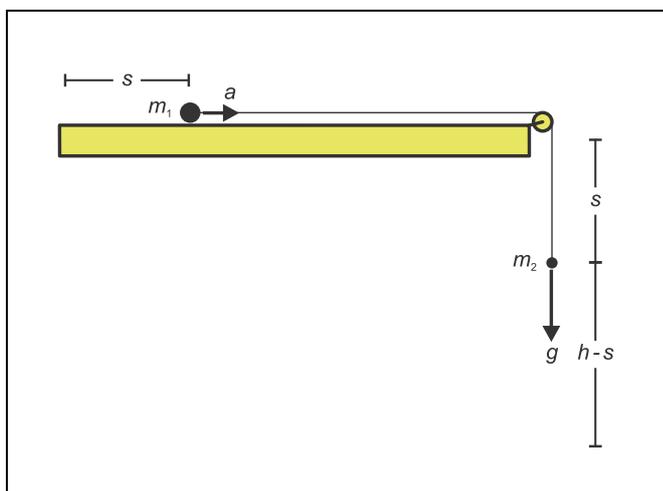


Fig. 1 Diagram illustrating a uniformly accelerated motion

Apparatus

1 linear air track	337 501
1 air supply for air track	337 53
1 power controller	337 531
1 VideoCom	337 47
1 plug-in unit 230 V / 12 V AC / 20 W	562 791
1 camera tripod	300 59
1 stand rod 10 cm	300 40
1 metal scale	311 02

additionally required:

1 PC with Windows 95 / 98 / NT

A computer program for VideoCom represents the entire motion of the slider as a path-time diagram and makes possible further evaluation of the measured values. In particular, computation of the velocity and of the acceleration can be activated with a mouse click. This enables the kinetic and potential energy to be computed and compared with each other.

Setup

The experimental setup is illustrated in Fig. 2.

- Mount the track rail on the track stand, set it up, and align it horizontally with the adjusting screws (see instruction sheet of the linear air track) using a spirit level.
- Plug the adapter for air supply **(a)** into the air inlet.
- Connect the air supply to the power controller; connect the tubing to the adapter for air supply (see instruction sheet of linear air track).
- Mount the guide pulley with small moment of inertia **(h)** to the stand rod 10 cm at the end of the linear air track.
- Attach the holding magnet with a clamping rider **(c)** near the air inlet, and put the brake **(g)** onto the other end of the track.
- Switch the air supply on, put the slider on the linear air track, and readjust the track with the adjusting screws until the slider remains at rest at several places of the track rail; the air flow should be varied until the parameters are optimised.

Setting up VideoCom:

- Screw VideoCom onto the camera tripod, set it up at a distance of approx. 2 m from the linear air track, and align it in height with the linear air track parallel to the track rail.
- Supply VideoCom with power via the plug-in unit, and connect it to a serial input of the PC (e.g. COM1).
- Connect the holding magnet to VideoCom via two connection leads (see instruction sheet of VideoCom).
- If necessary, install the VideoCom software on a PC with Windows 95/98/NT, call the program "VideoCom Motions" and, if necessary, choose the desired language and the serial interface (see instruction sheet of VideoCom).

Aligning VideoCom:

- Equip two sliders with interrupter flags **(d)** and stick retro-reflecting foil on both of them.
- Move the first slider to the holding magnet, and put the second slider on the linear air track while the air supply is switched off so that the distance between the two interrupter flags is exactly 1 m.
- Click "Intensity Test" in the program "VideoCom Motions".
- Slightly darken the room in order to minimise the background.
- Align VideoCom so that two peaks are visible on the LC display on the housing of the camera or on the screen respectively.
- Get rid of interfering light or reflections so that no other peaks are visible.
- Improve the alignment further until the ratio between the peaks and the background is greater than 5:1 for both sliders.

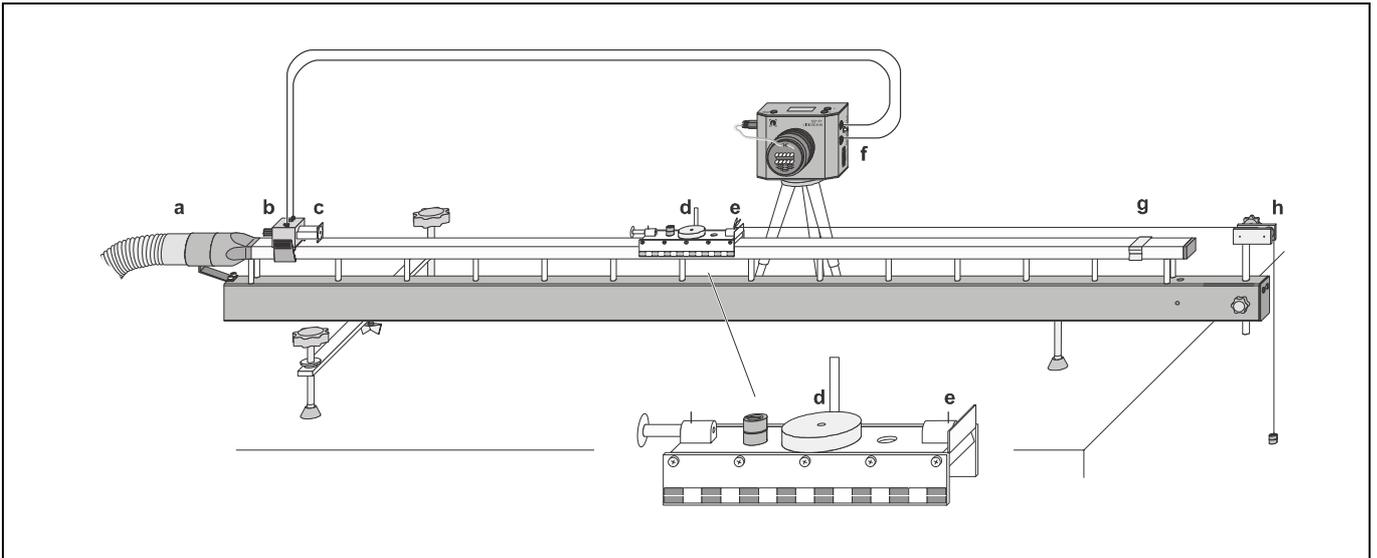
Connecting the holding magnet:

- Remove the second slider.
- Attach a 1.20 m long piece of thread to the thread holder **(e)** of the first slider, guide it around the pulley and suspend a 1 g weight from it.
- Switch the air supply on, and optimise the air flow.
- Equip the slider with the holding plate and move it to the holding magnet.
- Set the supply voltage for the holding magnet to its maximum value with the adjusting pin **(f)** at the VideoCom housing.
- Adjust the iron core of the holding magnet with the knurled screw **(b)** so that the slider is just kept and starts running immediately after pressing the key START at the VideoCom housing.

Compensating the distortion:

- Change to the representation "Path" in the program "VideoCom Motions".
- Equip the slider with both interrupter flags (distance = 5 cm).
- Call the menu "Settings/Path Calibration" with the button  or the key F5.
- Enter the values 0 m and 0.05 m as positions of the two interrupter flags in the register "Path Calibration".
- Click the button "Read Pixels From Display" and activate "Use Calibration".
- Call the menu "Settings/Path Calibration" anew and enter the following settings in the register "Setpoint Selection".

Δt	100 ms (10 fps)
Flash	Auto
Smoothing	Standard ($4 \cdot \Delta t$)
Stop measurement	after covering the path
s	e.g. 0.9 m
- Start the measurement with the button  or the key F9, and record the motion of the slider.
- Next click the button "Suggest Linearisation" in the register "Linearisation" of the menu "Settings/Path Calibration".



If an angle $\alpha \neq 0^\circ$ is displayed, the angle between the linear air track and VideoCom is not yet correct (see Fig. 3):

- Reject the linearisation with the button "Interrupt".
- Adjust the position of the linear air track by displacing the "right foot".
- Delete the old measured values with the button  or the key F4, record the motion of the slider, and determine the angle α anew.
- Repeat the procedure until $\alpha = 0^\circ$ is displayed; then activate "Use Linearisation" and take on the displayed distortion δ .

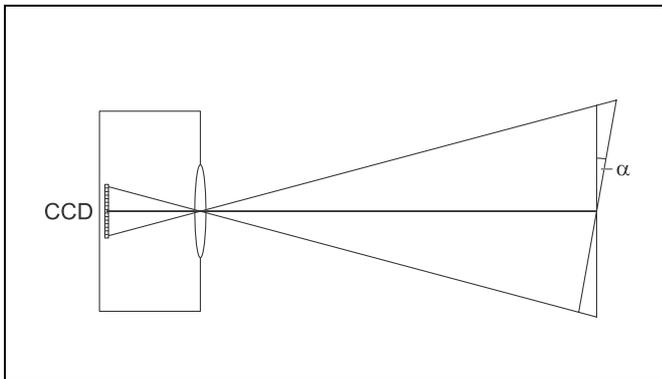


Fig. 3 Scheme of the definition of the angle α between the linear air track and VideoCom.

Path calibration:

- Equip both sliders with interrupter flags once more, move the first slider to the holding magnet, and put the second slider on the linear air track while the air supply is switched off so that the distance between the two interrupter flags is exactly 1 m.
- Enter the values 0 m and 1 m as positions of the two sliders in the register "Path Calibration" of the menu "Settings/Path Calibration".
- Click the button "Read Pixels From Display", and activate "Use Calibration".

Fig. 2 Experimental setup for recording motion with VideoCom. Here a uniform acceleration with the accelerated mass $m = 100$ g is shown.

Carrying out the experiment

a) Uniformly accelerated motion in dependence on the accelerated mass m :

- Delete old measured values with  or F4.
- Suspend two 1 g weights from the thread.
- Equip the slider, e.g., with the impact plate and two 1 g weights so that the total mass m of the slider and the two 1 g weights suspended from the thread is 100g (see instruction sheet of the linear air track).
- Move the slider to the holding magnet, start the measurement with  or F9, and record the path-time diagram.
- Put an additional 100 g weight on the slider and record another path-time diagram.
- Record path-time diagrams with a second and a third additional 100 g weight.
- Store the measured values with  or F2 (use a filename that allows you to recognise the file).

b) Uniformly accelerated motion in dependence on the accelerating force F :

- Delete old measured values with  or F4.
- Remove the additional 100 g weights from the slider, suspend one 1 g weight from the thread, and put three 1 g weights on the slider (total mass $m = 100$ g).
- Move the slider to the holding magnet, start the measurement with  or F9, and record the path-time diagram.
- Remove one 1 g weight from the slider, suspend it from the thread, and record the path-time diagram with two accelerating 1 g weights.
- Record the path-time diagram with three and four 1 g weights suspended from the thread.
- Store the measured values with  or F2 (use a filename that allows you to recognise the file).

Measuring example and evaluation

a) Uniformly accelerated motion in dependence on the accelerated mass m :

- Fig. 4 shows the path-time diagram for different accelerated masses m and the same accelerating force $F = m_2 \cdot g$. The path-time curves are branches of parabolas, the origin being the vertex. At a greater mass, the slider needs more time for covering the same path length $s = 1$ m because the acceleration is smaller in this case.

Computing the kinetic energy:

- Call the menu "Settings/Path Calibration" with the button  or the key F5.
- Click the register "Formula" and make the following entries:
 Quantity: Energy
 Symbol: E_{kin}
 Unit: mJ
 Minimum: -1 (mJ)
 Maximum: 40 (mJ)
 Formulas: $0.5 \cdot 100 \cdot v^2$
 $0.5 \cdot 200 \cdot v^2$
 $0.5 \cdot 300 \cdot v^2$
 $0.5 \cdot 400 \cdot v^2$
- Confirm the entries with "OK", and click the new register "Energy" (see Fig. 5).

Computing the potential energy:

- Make the following changes in the register "Formula":
 Symbol: E_{pot}
 Formulas: $2 \cdot 9.81 \cdot (1-s)$
 $2 \cdot 9.81 \cdot (1-s^2)$
 $2 \cdot 9.81 \cdot (1-s^3)$
 $2 \cdot 9.81 \cdot (1-s^4)$
- Confirm the entries with "OK", and click the new register "Energy" (see Fig. 6).

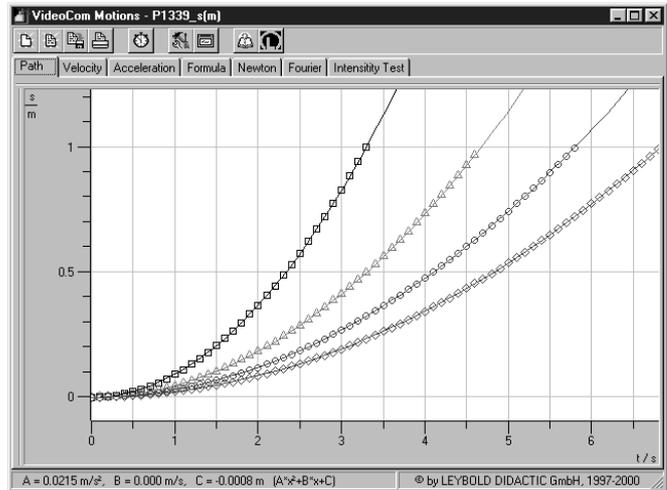


Fig. 4 Path-time diagram of uniformly accelerated motions for $m = 100$ g (\square), 200 g (\triangle), 300 g (\circ) and 400 g (\diamond) (accelerating mass $m_2 = 2$ g)

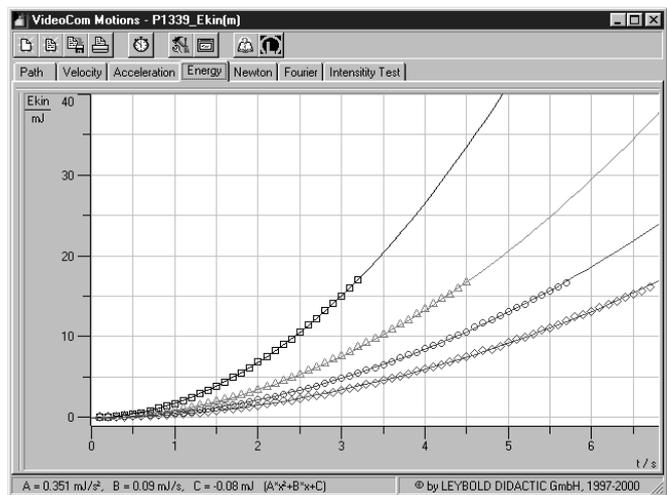


Fig. 5 Kinetic energy of uniformly accelerated motions as a function of time; accelerated masses $m = 100$ g (\square), 200 g (\triangle), 300 g (\circ) and 400 g (\diamond) (accelerating mass $m_2 = 2$ g)

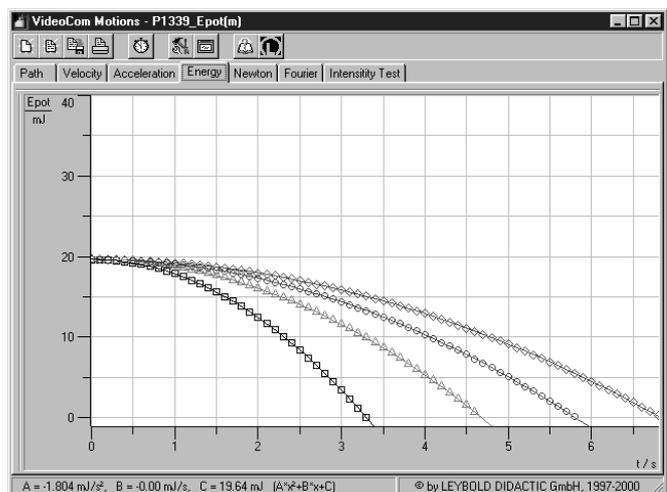


Fig. 6 Potential energy of uniformly accelerated motions as a function of time; accelerated masses $m = 100$ g (\square), 200 g (\triangle), 300 g (\circ) and 400 g (\diamond) (accelerating mass $m_2 = 2$ g)

Computing the total energy:

- Make the following changes in the register “Formula”:
 Symbol: E
 Formulas: $0.5 \cdot 100 \cdot v_1^2 + 2 \cdot 9.81 \cdot (1 - s_1)$
 $0.5 \cdot 200 \cdot v_2^2 + 2 \cdot 9.81 \cdot (1 - s_2)$
 $0.5 \cdot 300 \cdot v_3^2 + 2 \cdot 9.81 \cdot (1 - s_3)$
 $0.5 \cdot 400 \cdot v_4^2 + 2 \cdot 9.81 \cdot (1 - s_4)$
- Confirm the entries with “OK”, and click the new register “Energy” (see Fig. 7).

Fig. 5 shows the kinetic energy, Fig. 6 the potential energy and Fig. 7 the total energy of the uniformly accelerated motion for different accelerated masses as functions of the time t .

The kinetic energies are all located on branches of parabolas, the origin being the vertex. This behaviour is in agreement with Eqs. (IV) and (V), from which it follows that:

$$E_{\text{kin}} = \frac{1}{2} \cdot m \cdot a^2 \cdot t^2 \quad (\text{X})$$

The potential energies form dropping branches of parabolas, the vertex being determined by the maximum potential energy. The maximum potential energy is the same in all cases. This behaviour can be derived from Eqs. (II) and (VIII):

$$E_{\text{pot}} = m_2 \cdot g \cdot \left(h - \frac{1}{2} \cdot a \cdot t^2 \right) \quad (\text{XI})$$

After covering the path $s = 1$ m, all masses attain the same kinetic and potential energies to a good approximation.

Because of $m_2 \cdot g = m \cdot a$,

$$E = E_{\text{kin}} + E_{\text{pot}} = m_2 \cdot g \cdot h \quad (\text{XII})$$

is obtained from Eqs. (X) and (XI). Fig. 7 shows, indeed, that the total energy is constant to a good approximation at small velocities. At greater velocities frictional losses play an increasingly important role.

b) Uniformly accelerated motion in dependence on the accelerating force F :

- Fig. 8 shows the parabola-shaped path-time diagrams of the slider for different accelerating forces $F = m_2 \cdot g$ with the same accelerated mass m . When the force is greater, the slider covers the path length $s = 1$ m in a shorter time interval because the acceleration is greater in this case.

Computing the kinetic energy:

- Click the register “Formula” and make the following entries:
 Quantity: Energy
 Symbol: E_{kin}
 Unit: mJ
 Minimum: -1 (mJ)
 Maximum: 40 (mJ)
 Formulas: $0.5 \cdot 100 \cdot v_1^2$
 $0.5 \cdot 100 \cdot v_2^2$
 $0.5 \cdot 100 \cdot v_3^2$
 $0.5 \cdot 100 \cdot v_4^2$
- Confirm the entries with “OK”, and click the new register “Energy” (see Fig. 9).

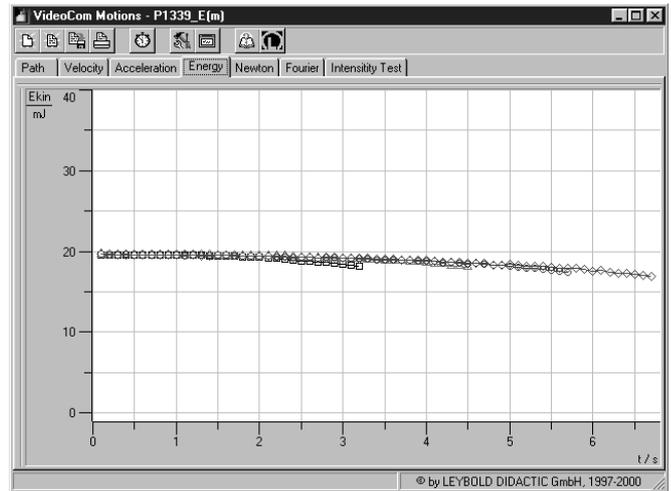


Fig. 7 Total energy of uniformly accelerated motions as a function of time; accelerated masses $m = 100$ g (\square), 200 g (\triangle), 300 g (\circ) and 400 g (\diamond) (accelerating mass $m_2 = 2$ g)

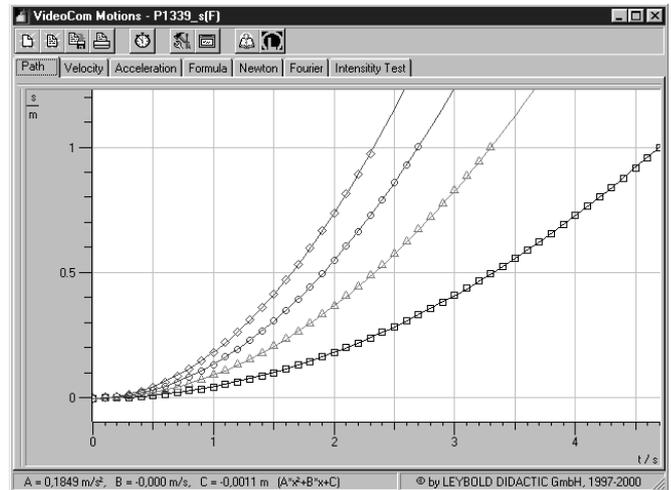


Fig. 8 Path-time diagram of the uniformly accelerated motion for the accelerating masses $m_2 = 1$ g (\square), 2 g (\triangle), 3 g (\circ) and 4 g (\diamond) (accelerated mass $m = 100$ g)

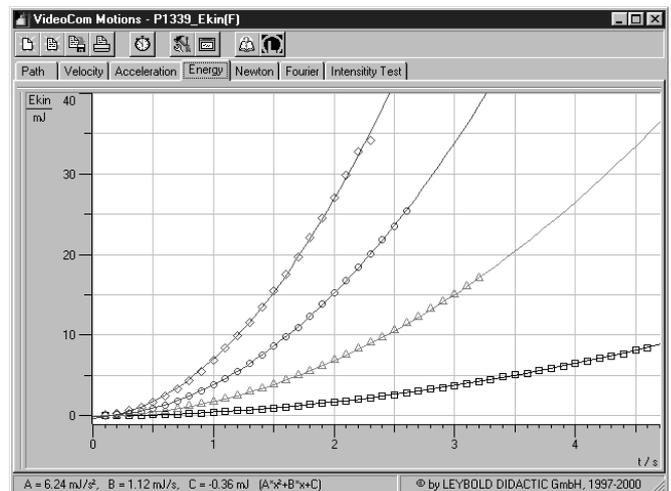


Fig. 9 Kinetic energy of uniformly accelerated motions as a function of time; accelerating masses 1 g (\square), 2 g (\triangle), 3 g (\circ) and 4 g (\diamond) (accelerated mass $m = 100$ g)

Computing the potential energy:

- Make the following changes in the register "Formula":

Symbol: E_{pot}
 Formulas: $1 \cdot 9.81 \cdot (1-s_1)$
 $2 \cdot 9.81 \cdot (1-s_2)$
 $3 \cdot 9.81 \cdot (1-s_3)$
 $4 \cdot 9.81 \cdot (1-s_4)$

- Confirm the entries with "OK", and click the new register "Energy" (see Fig. 10).

Computing the total energy:

- Make the following changes in the register "Formula":

Symbol: E
 Formulas: $0.5 \cdot 100 \cdot v_1^2 + 1 \cdot 9.81 \cdot (1-s_1)$
 $0.5 \cdot 100 \cdot v_2^2 + 2 \cdot 9.81 \cdot (1-s_2)$
 $0.5 \cdot 100 \cdot v_3^2 + 3 \cdot 9.81 \cdot (1-s_3)$
 $0.5 \cdot 100 \cdot v_4^2 + 4 \cdot 9.81 \cdot (1-s_4)$

- Confirm the entries with "OK", and click the new register "Energy" (see Fig. 11).

Fig. 9 shows the kinetic energy, Fig. 10 the potential energy and Fig. 11 the total energy of the uniformly accelerated motion for different accelerating forces as functions of the time *t*.

In agreement with Eq. (X), the kinetic energies are located on the branches of parabolas, the origin being the vertex. The potential energies form dropping branches of parabolas, in accordance with Eq. (XI), the vertex being determined by the maximum potential energy, which depends on the accelerating force. At low velocities, the total energy is constant to a good approximation.

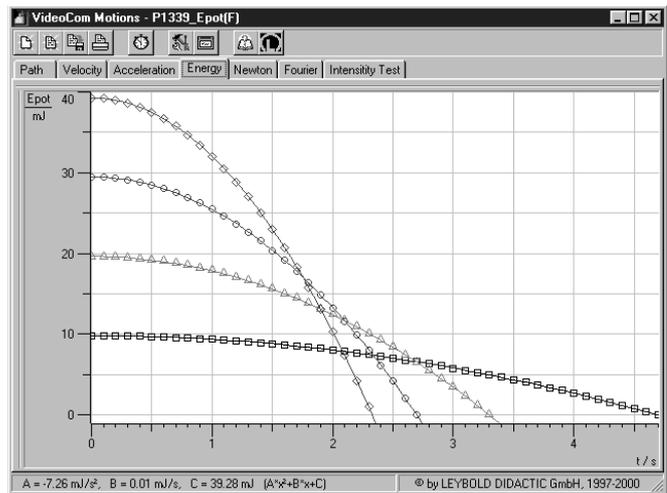


Fig. 10 Potential energy of uniformly accelerated motions as a function of time; accelerating masses 1 g (□), 2 g (Δ), 3 g (○) and 4 g (◇) (accelerated mass *m* = 100 g)

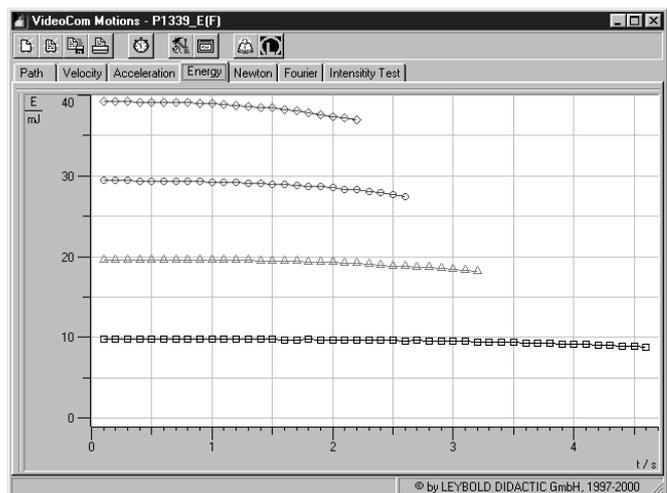


Fig. 11 Total energy of uniformly accelerated motions as a function of time; accelerating masses 1 g (□), 2 g (Δ), 3 g (○) and 4 g (◇) (accelerated mass *m* = 100 g)