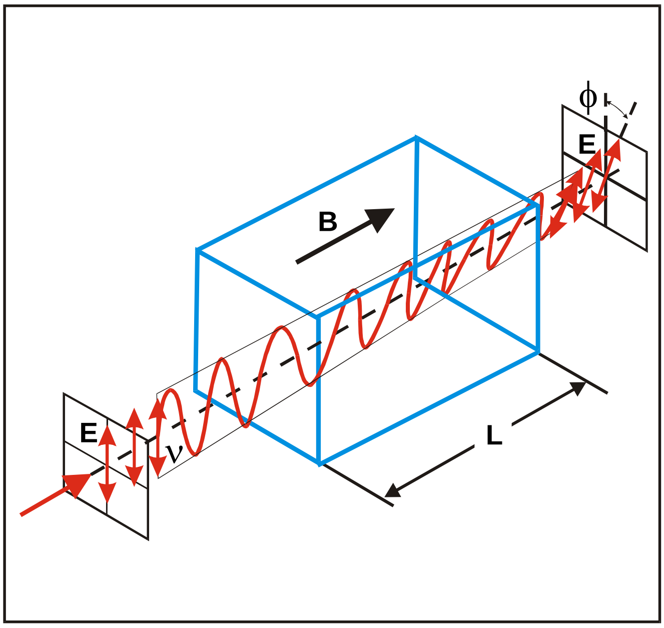
Farady effect: Determining Verdet's constant for flint glass as a function of the wavelength

Objects of the experiment

- Observing the rotation of the polarization plane if polarized monochromatic light is passing a flint glass in a magnetic field
- Determining Verdet’s constant from the relation between rotation angle and magnetic flux.
- Verification of the relationship between Verdet’s constant and wavelength.

Fig. 1: Faraday effect schematically: B magnetic flux density parallel to the propagation direction of the polarized light. E: electric field, L: length of the isotropic material.



Principles

In 1845 Faraday discovered the following phenomena: If a transparent isotropic materials is placed in a strong magnetic field and linearly polarized light is transmitted in the direction of the magnetic field the plane of polarization of linearly polarized light rotates by an angle ϕ when passing through the transparent material (Fig. 1). The angle of rotation ϕ is proportional to the magnetic flux density B and the length L of the medium through which the light is transmitted:

$$\phi = V \cdot B \cdot L \qquad \text{(I)}$$

L: length of the transparent material

B: magnetic flux density

V: Verdet’s constant

The proportionality constant V is called Verdet’s constant. V depends on the wavelength λ of the light and the temperature.

This observation can be explained by imaging linearly polarized light as the coherent superposition of two opposite polarized components σ_+ and σ_-.

In atomic physics the magnetic field influences the motion of the electrons in the atom. The magnetic field causes the oscillating charges to carry out an additional precession movement. The precession frequency is equal to the Lamor frequency:

$$\omega_L = \frac{e}{m} \cdot B \qquad \text{(II)}$$

e: chare of the oscillating particles

m: mass of the oscillating particles

B: magnetic flux density

The components σ_+ and σ_- of the polarized light have different frequencies relative to the charges performing a precession movement.

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One component has the frequency $\omega + \omega_L$ the other component has the frequency $\omega - \omega_L$. The refractive indices n_+ and n_- and the phase velocities v_+ and v_- differ which is equivalent to optical activity.

The following equation can be derived for the angle ϕ of rotation of the polarization plane as function of the length L of the material through which the light is transmitted:

$$\phi = \frac{\omega \cdot (n_+ - n_-)}{2\,c} \cdot B \qquad \text{(III)}$$

ω: frequency of the transmitted light

n_+, n_-: refractive indices

c: velocity of light

If the refractive index n is known as a function of the wavelength λ the Verdet's constant can be calculated using the assumptions above:

$$V = \frac{e\,\lambda}{2\,m\,c^2} \cdot \frac{dn}{d\lambda} \qquad \text{(IV)}$$

e: charge of electron

m: mass of electron

c: velocity of light

Equation (IV) holds for many spectral ranges. In this experiment flint glass is used. For this type of glass the following approximation applies:

$$\frac{dn}{d\lambda} = -\frac{1.8 \cdot 10^{-14}}{\lambda^3}\,m^3 \qquad \text{(V)}$$

With this equation follows that the Verdet's constant should decrease with the wavelength as $\frac{1}{\lambda^2}$:

$$V = -\frac{e}{2mc^2}\,\frac{1.8 \cdot 10^{-14}}{\lambda^2}\,m^2 \qquad \text{(VI)}$$

The relationship $\frac{e}{m}$ can be derived from purely optical measurements and with the knowledge of the velocity of light. The values obtained for $\frac{e}{m}$ aggress well for some materials with known values. This shows that the natural oscillations of the electrons are actually responsible for the Faraday effect.

In this experiment a homogeneous magnetic field of sufficient strength is not available to enable to verify the equations (I) and (IV) quantitatively. We will therefore restrict ourselves to

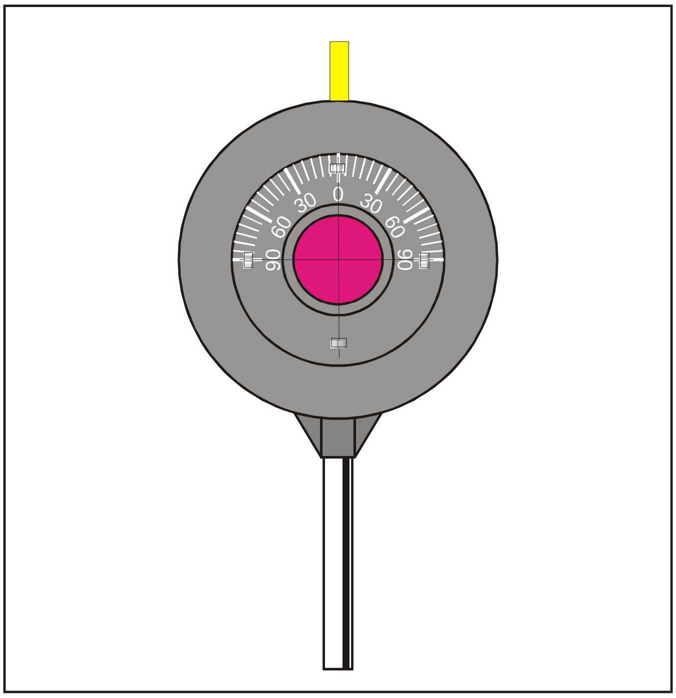
a) verifying the proportionality between the rotation of the polarization plane ϕ and the magnetic field B.

b) demonstrating the decrease of the Verdet's constant V with increasing wavelength λ.

Apparatus

1 Flint glass square with holder	560 482
1 Rider base with threads	460 381
1 U-core with yoke	562 11
1 Pair of bored pole pieces	560 31
2 Coil with 250 turns	562 13
1 Halogen lamp, 12 V / 90 W	450 63
1 Halogen lamp housing 12 V, 50/90 W	450 64
1 Transformer 2 to 12 V; 120 W	521 25
1 Picture slider	450 66
1 Monochromatic filter, yellow	468 05
1 Monochromatic filter, blue-green	468 09
1 Monochromatic filter, blue-violet	468 11
1 Monochromatic filter, violet	468 13
1 Lens in frame f = +50 mm	460 02
2 Polarization filter	472 401
1 Translucent screen	441 53
5 Optics rider 60/50	460 373
1 Optical bench, standard cross section 1 m	460 32
1 Variable extra low-voltage transformer	521 39
1 Digital-analog multimeter METRAHit 24 S	531 281
1 Universal Measuring Instrument Physics	531 835
or	
1 Mobile-CASSY	524 009
1 Combi B-Sensor S	524 0381
1 Extension cable, 15-pole	501 11
1 Stand base, V-shape, 20 cm	300 02
1 Stand rod, 25 cm	300 41
1 Leybold multiclamp	301 01
1 Pair cables 50 cm, red/blue	501 45
1 Pair cables 100 cm, red/blue	501 46

Fig. 2: Attaching a thread cross to the analyzer.



Setup

In this experiment a thread cross is attached to the analyzer and projected onto the translucent screen so that the angle of rotation $\Delta\varphi$ can be determined precisely.

Note: The measuring method can be improved when the thread cross is projected onto a separate angular scale with degree divisions (e.g. drawn on a sheet of paper or 459 40) which is attached to the translucent screen. All analyzer settings can be read off from this angular scale easily.

- Equip one of the polarizing filters with a thread cross like depicted in Fig. 2. Make sure, that the cross is exactly at right angles and placed exactly in the center of the analyzer. Use the analyzer protractor scale to align it. The best material for this is silk thread.

The experimental setup is shown in Fig. 3.

- Arrange the halogen lamp on the optical bench according to picture (Fig. 3). Mount the picture slider with heat insulation filter on the condenser.
- Position a polarizer close the halogen lamp on the optical bench as shown in Fig. 3.
- Place the U-core of the demountable transformer with the two coils on the rider base with thread and fix it with the holder for the flint glass square.
- Place the bored pole pieces on the U-core in such a manner that the flint glass square can be placed on the holder as depicted in Fig. 3.
- Push the pole pieces right up to the flint glass square but without touching it.

Safety notes

Mind the instruction sheet 450 64 for operating the halogen lamp.

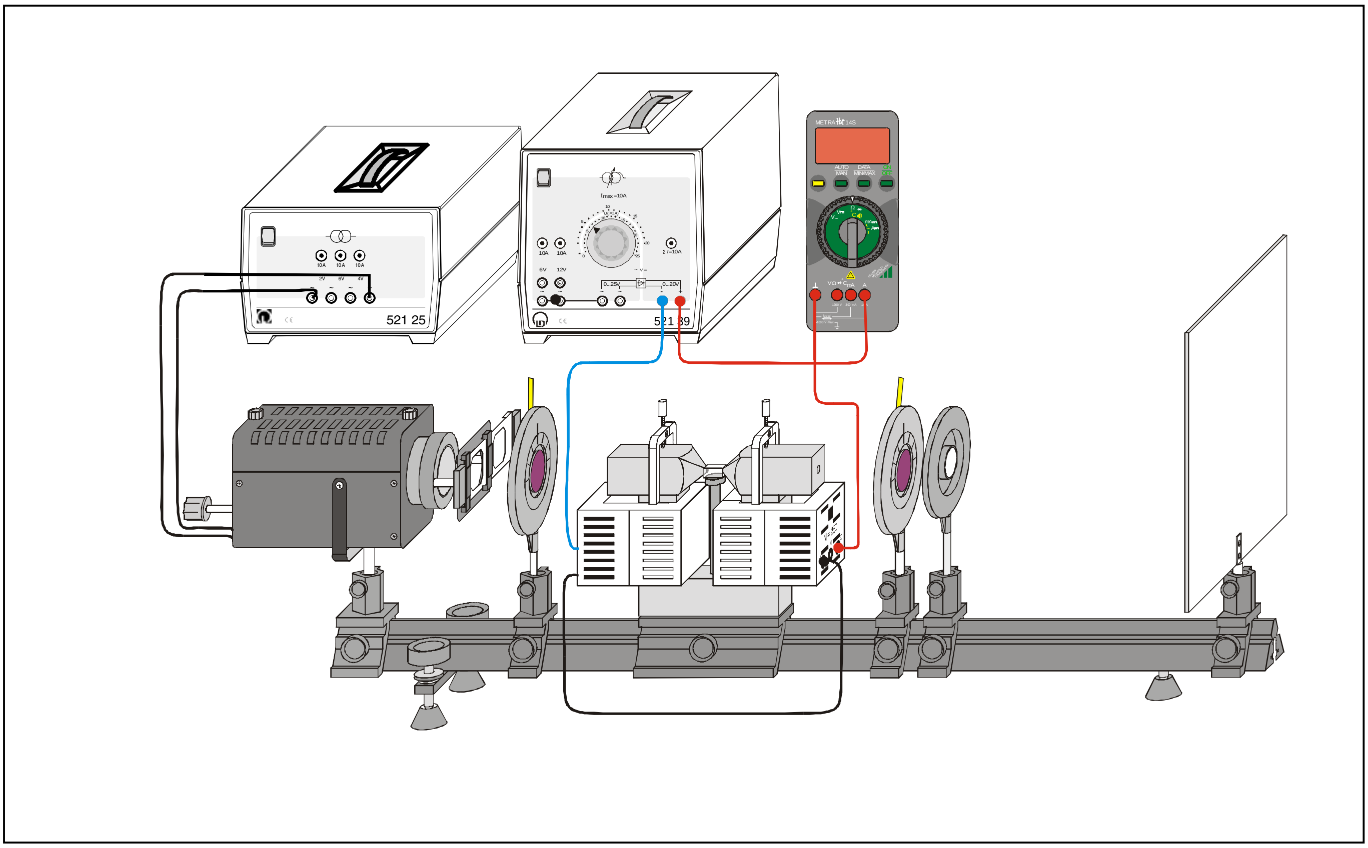
- Operate with AC voltage only (i.e. 12 V AC)
- Do not cover the ventilation slits (danger of overheating).
- Do not touch the bulb of the halogen lamp with your fingers.
- Protect the heat filter of the picture slider from shocks, dropping or similar (the fragile material can break easily).

- Use the clamps to fix the bored pole pieces on the U-core.
- Position the analyzer (polarizer) close the U-core on the optical bench as shown in Fig. 3.
- Position the translucent screen opposite to the analyzer.
- Place the lens f = +50 mm between the analyzer and the translucent screen.

Electrical setup

- Connect the coils and the ammeter (digital multimeter) in series to the variable extra low-voltage transformer.
- Connect the halogen lamp to 12 V sockets of the transformer 120W.

Fig. 3: Experimental setup to investigate the rotation of the polarization plane in a magnetic field.



Optical adjustment

- Remove the housing cover with ventilation slits of the halogen lamp housing. Use the 100-W lamp with reflector in the halogen lamp – for detailed assembly hints see instruction sheet 450 64.
- User picture slider with the heat filter for absorbing the infrared component in halogen light.
- Switch on halogen lamp and form an image of the lamp coil on the translucent screen using the condenser.
- Align the light source and the bored pole pieces in such a manner that light – as much as possible – passes through the bores in the pole pieces (with no flint glass square on the holder).
- To project an image of the thread cross on the analyzer onto the translucent screen shift the lens towards the analyzer until a sharp image is observed.

Note: When using an angular scale on the translucent screen the thread cross and protractor scale should be exactly concentric.

- Insert the polarizing filter.

Carrying out the experiment

a) Calibration of the magnetic field

- Remove the flint glass square.
- Connect the Combi B-Sensor S to the Universal Measuring Instrument Physics or the Mobile CASSY using the extension cable.
- Place the tangential B probe of the Combi B-Sensor S between the pole pieces as shown in Fig. 4. Use the stand material to hold the magnetic probe between the bored pole pieces.
- Record the magnetic field B as function of the current I through the coils.

b) Rotation of the polarization plane ϕ as function of the magnetic field B

- Insert the filter with λ = 450 nm (468 11) in the diagram slider.
- Align the flint glass square between the bored pole pieces.
- Set the desired magnetic field by means of the magnet current.
- Set the analyzer to 0° position.
- Find the intensity minimum by turning the polarizer.

Note: For a final minimum adjustment (almost dark) the minimum light intensity can be easily checked by looking directly into the light beam behind the imaging lens. The polarizer and analyzer are set to the intensity minimum as it can be easier accessed then the intensity maximum.

- Reverse the polarity of the magnetic field without changing the magnet current. To do so, switch off magnetic field; reverse polarity of coil current; switch on magnetic field.

Note: When the direction of the field is reversed the doubled angle of rotation 2ϕ is measured (Fig. 5).

- Find the intensity minimum by turning the analyzer.
- Switch off the magnetic field and find the intensity maximum by turning the polarizer.

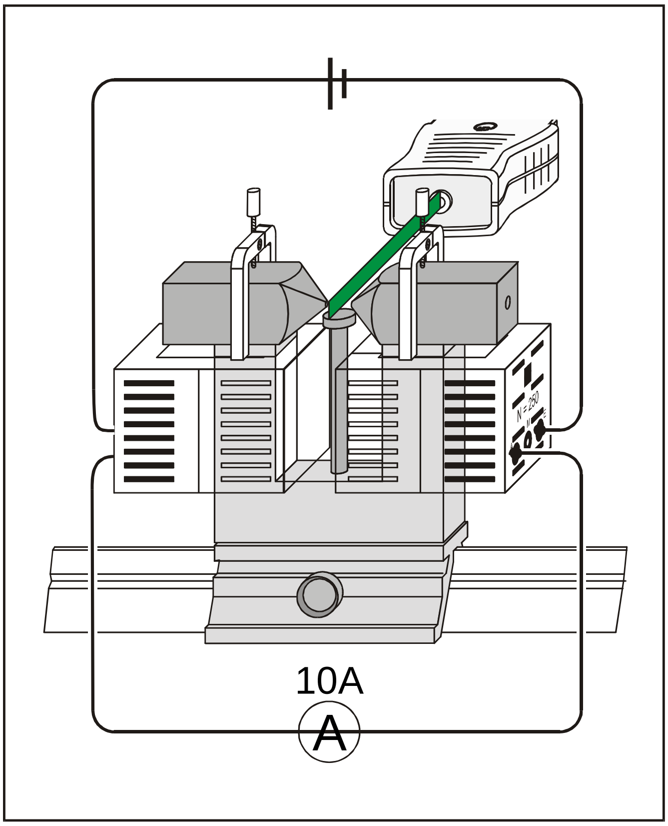
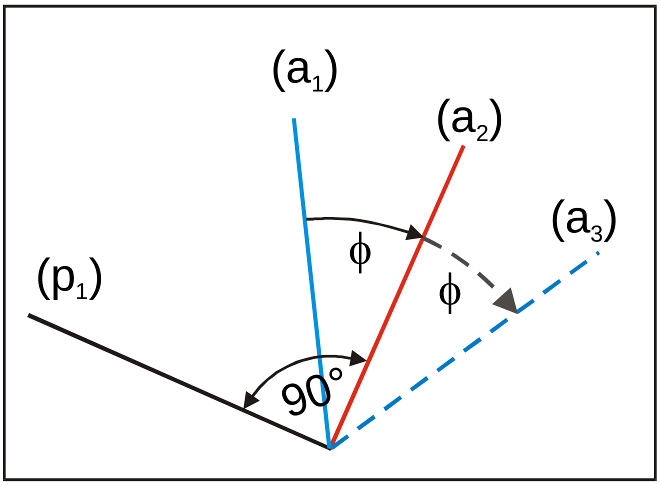


Fig. 4: Calibration of the magnetic field schematically.

- Remove the color filter from the beam path to enhance the contrast of the projected thread cross.
- Read off the thread cross position.
- Repeat this measurement steps for various magnetic fields by varying the magnetic current.

Fig. 5: Measuring the doubled angle of rotation 2ϕ by reversing the polarity of the magnetic field.
ϕ: angle of rotation of the polarization plane
(a_1): analyzer position at the start
(a_2): polarization plane of the light after passing through the flint glass square in both experiments.
(p_1) polarizer position for the first magnetic field setting.
(a_3): analyzer position after reversing the polarity of the magnetic field.



c) Verdet's constant as function of the wavelength λ

- Place a color filter with the desired wavelength in the picture slider of the halogen lamp fitting.
- Set the thread cross to zero.
- Place the filter in the beam path and apply the maximum magnetic field.
- Measure the rotation of the polarization plane 2ϕ as described in paragraph b).
- Remove the color filter from the beam path before reading off the thread cross position.
- Repeat the measurement for the other filters.

Measuring example

a) Calibration of the magnetic field

Table. 1: Magnetic flux density B as function of the current I through the coils.

$\frac{I}{A}$	$\frac{B}{mT}$
1.0	26
2.0	52
3.0	78
4.0	107
5.0	132
6.0	159
7.0	182
8.0	202
9.0	221
10.0	240

b) Rotation of the polarization plane ϕ as function of the magnetic field B

Table. 2: Rotation angle 2ϕ for constant wavelength λ = 450 nm (filter: blue with violet) as function of the current I and magnetic flux density B, respectively.

$\frac{I}{A}$	$\frac{B}{mT}$	$\frac{2\phi}{°}$
1.0	26	2
2.0	52	3
3.0	78	4
4.0	107	6
5.0	136	8
6.0	159	9
7.0	182	10
8.0	202	11
9.0	221	12
10.0	240	13

c) Verdet's constant as function of the wavelength λ

Table. 3: Rotation angle 2ϕ as function of the wavelength λ at maximum magnetic flux density B= 240 mT. The wavelength is taken as the middle of the band pass of the color filter.

Filter	transmitted color	$\frac{\lambda}{nm}$	$\frac{2\phi}{°}$
468 05	yellow	570	8.5
468 09	blue-green	515	10
468 11	blue with violet	450	14
468 13	violet	440	15

Evaluation and results

a) Calibration of the magnetic field

Fig. 6 summarizes the results of table 1.

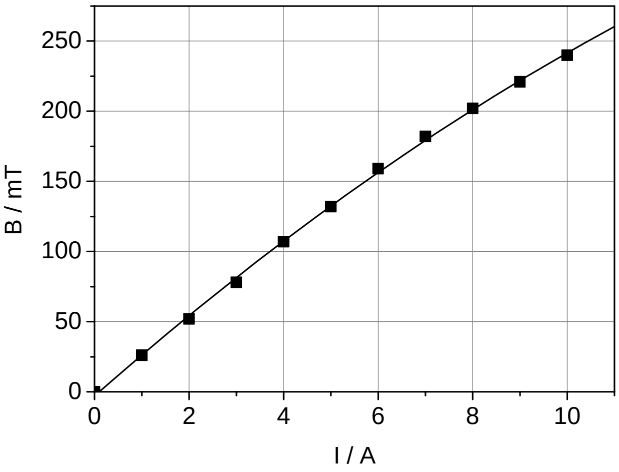


Fig. 6: Calibration curve: Relationship between the current I trough the coils and the magnetic field B.

b) Rotation of the polarization plane ϕ as function of the magnetic field B

The linear proportionality between the rotation of the polarization plane 2ϕ and magnetic field B can clearly be seen in Fig. 7. Verdet's constant can be determined from the fit of a straight line to the experimental data. For blue light (λ = 450 nm) the slope is determined to

$$V \cdot L = \frac{2\phi}{2B} = \frac{\phi}{B} = 0.0551\frac{\mathrm{deg}}{\mathrm{T}}$$

With the length of the flint glass square: L = 0.02 m

we obtain for the Verdet's constant:

$$V = \frac{\phi}{BL} = 1377\frac{\mathrm{deg}}{\mathrm{T\,m}}$$

c) Verdet's constant as function of the wavelength λ

The decrease in the angle of rotation 2ϕ with increasing wavelength λ can be seen in Fig. 8. If rotation of the polarization plane 2ϕ is plotted as function of λ^{-2} (Fig. 9) we obtain the expected proportionality in accordance with equation (VI).

The results of b) and c) verifies the dependence of the Faraday effect on the magnetic field and wavelength.

Supplementary information

Historically, Faraday's observaiotion was the first experimental evidence that light and magnetism are related. Some years later Maxwell's theory described light as an electromagnetic wave.

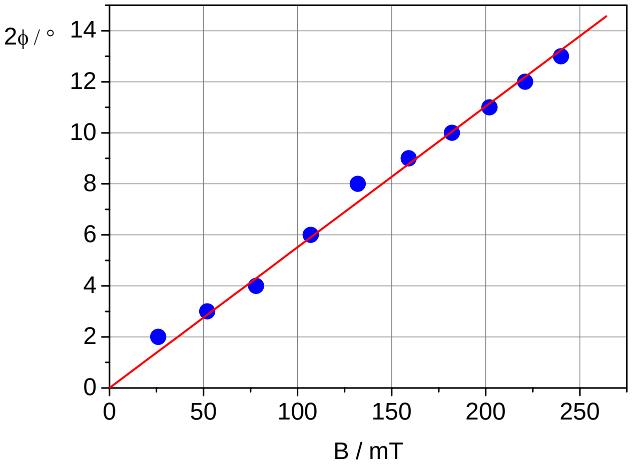


Fig. 7: Rotation of the polarization plane 2ϕ as function of the magnetic field B for the color filter blue with violet (λ = 450 nm).

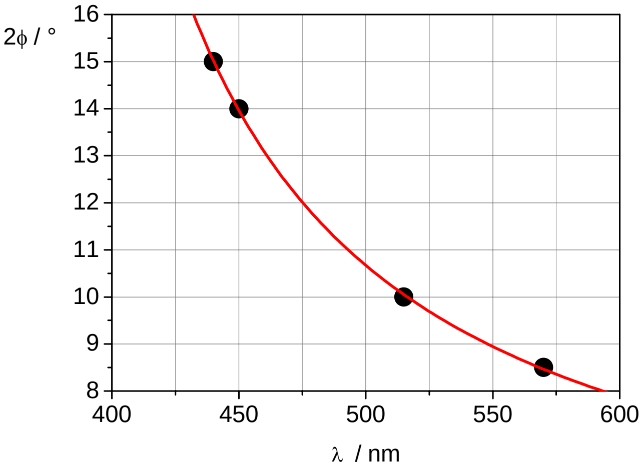


Fig. 8: Rotation of the polarization plane 2ϕ as function of the wavelength λ.

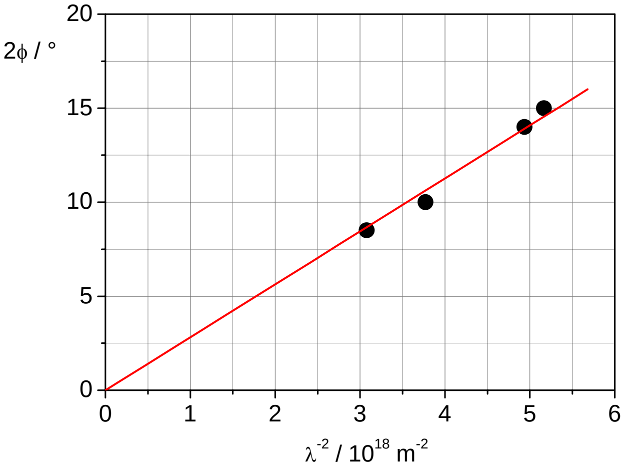


Fig. 9: Rotation of the polarization plane 2ϕ as function of λ^{-2}