

Quarter-wavelength and half-wavelength plate

Objects of the experiment

- Measuring the light intensity as function of the analyzer position.
- Using the quarter wave plate to produce circularly polarized light.

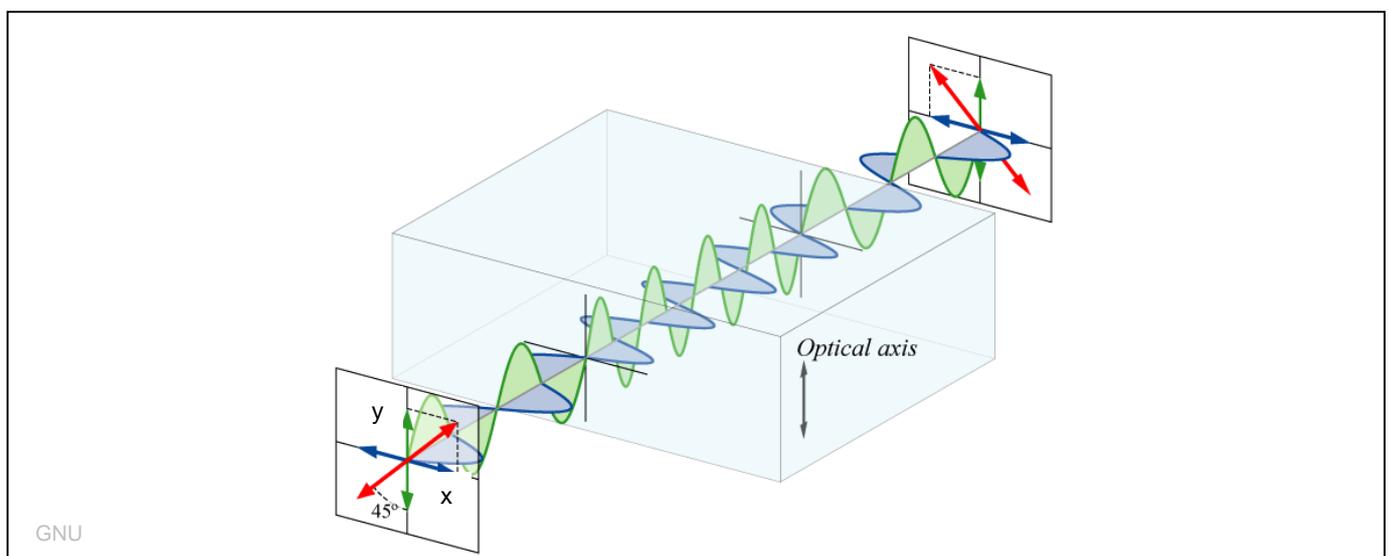
Principles

A wave plate or retarder is an optical device that alters the polarization state of a light beam travelling through it. A typical wave plate is simply a birefringent crystal or a double refracting plastic foil with a carefully chosen thickness.

If a beam of parallel light strikes perpendicularly a wave plate the light beam is splitted into two components due to its double refracting properties. The two components have planes of oscillation perpendicular to each other and slightly different phase velocities. For a quarter-wave plate the thickness of the foil is chosen in such a manner that the light component whose electric field vector oscillates in parallel to the rotation lever lags by a $\lambda/4$ behind other perpendicular oscillating light component. For a half-wave plate the thickness is chosen so that the created phase difference has the amount of $\lambda/2$.

In this experiment monochromatic light falls on a quarter-wave and half-wave plate. The polarization of the emergent light is investigated at different angles between the optic axis of the wave plates and the direction of the incident light.

Fig. 1: A half-wave plate schematically. Linearly polarized light entering a wave plate can be resolved into two waves, parallel (shown as green) and perpendicular (blue) to the optical axis of the wave plate. In the plate the parallel wave propagates slightly slower than the perpendicular one. At the far side of the plate the parallel wave is exactly half of a wavelength delayed relative to the perpendicular wave.



Apparatus

2 Quarter-wave plate	472 601
1 Half-wavelength plate	472 59
2 Polarization filter	472 401
1 Light filter yellow	468 30
1 Si Photocell STE 2/19	578 62
1 Holder for plug-in elements	460 21
1 Multimeter METRAHit Pro	531 282
1 Translucent screen	441 53
1 Optical bench, standard cross section 1 m	460 32
7 Optics rider 60/34	460 370
1 Halogen lamp housing 12 V, 50/90 W	450 64
1 Halogen lamp, 12 V / 90 W	450 63
1 Picture slider	450 66
1 Transformer 2 to 12 V, 120 W	521 25
2 Pair cables 100 cm, red/blue	501 46

Setup

The experimental setup is shown in Fig. 2 schematically.

Note: For the optical setup alternatively the small optical bench (460 43) or the optical bench S1 profile (460 310) can be used.

Notes on the beam path:

- The light supplied by the Halogen lamp (a) is concentrated by the condenser (b) and passes through a heat resistance filter to protect the optical components against heating up.
- Additionally, a heat protection filter filled with water might be used (in Fig. 2 indicated in dotted lines) to reduce the infrared radiation leading to a large background signal detected with the photo cell.

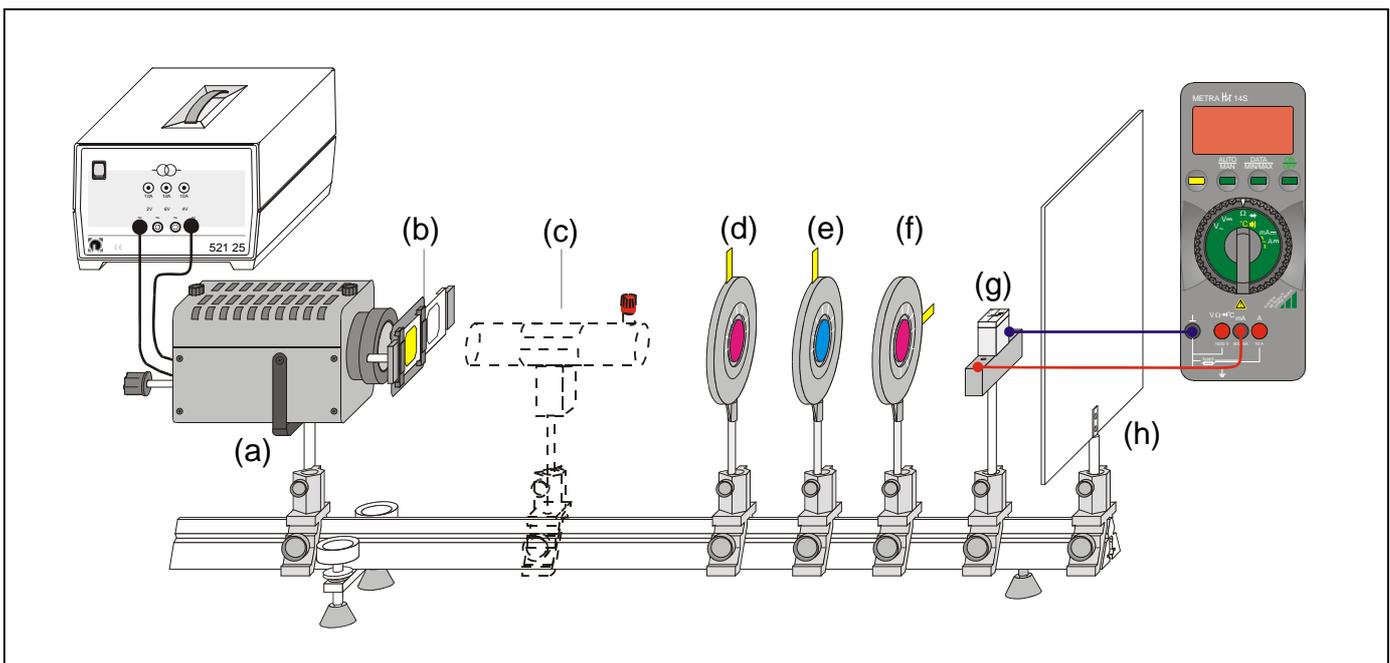
Optical adjustment:

- Set up the Halogen lamp (a) with the reflecting mirror and fit the condenser and picture slider in to the lamp housing.
- Insert the light filter yellow in front of the heat filter in the picture slider.
- Setup the polarizer, the $\lambda/4$ -wave plate and the analyzer like shown in Fig. 2 on the optical bench. The distance between the polarizer and the halogen lamp is about 20 cm to 30 cm.
- Setup the Si-photo cell behind the analyzer and adjust the path of the light ray that the photo cell is well illuminated.
- By turning the lamp insert in the halogen lamp housing the illumination can be adjusted. Produce a sharp image of the lamp coil on a small sheet of paper positioned at the center of the Si photo cell (g).

Note: The translucent screen depicted in Fig. 2 is used to perform the experiment qualitatively.

Fig. 2: Experimental setup for investigating the type of polarization of the emergent light (schematically).

- | | | |
|---------------------------------|---|------------------------|
| (a) halogen lamp | (d) polarizer | (g) Si photo cell |
| (b) picture slider with filters | (e) $\lambda/4$ or $\lambda/2$ wave plate | (h) translucent screen |
| (c) heat protection filter | (f) analyzer | |



Safety notes

Care should be taken that the various filters are not damaged by overheating.

- Don't place the *polarization filter* directly in front of the light source. Use a heat protection filter to prevent damage of the dichroic plastic foil from overheating.
- Don't place *quarter wave plate* or a *half wave plate* directly in front of a hot light source to prevent the double refracting foil from overheating.

- For measuring the photo current connect the Si-photo cell via the pair of cables red/blue to the multimeter.

Note: The photo current is proportional to the light intensity. The light intensity is proportional to the electric field vector to the square: $I \sim E^2$

Carrying out the experiment

a) Quarter wave plate

- Remove the quarter wave plate and set the polarizer to the zero position.
- Measure the light intensity as function of the analyzer position over the range -90° to 90° .
- Clamp the quarter wave plate into the optic rider between the polarizer and analyzer.
- Measure the light intensity as function of the position of the analyzer (i.e. angles 0° , 30° , 45° and 60°) over the range -90° to 90° .

b) Half wave plate

- Set the polarizer to the zero position.
- Clamp the half wave plate into the optic rider between the polarizer and analyzer.
- Measure the light intensity as function of the position of the analyzer (i.e. angles 0° , 30° , 45°) over the range -90° to 90° .

Note: The half wave plate can be replaced also by two quarter wave plates with same orientation.

Measuring example

Table 1 and Table 2 summarize the results. The inevitable background signal due to the infrared component has been baseline corrected.

a) Quarter wave plate

Table 1: Measured current as a function of the analyzer position α for different quarter wave plate positions φ . (Note: second column – measured without quarter wave plate).

position φ	–	0°	30°	45°	60°
$\frac{\alpha}{\text{deg}}$	$\frac{I}{\mu\text{A}}$	$\frac{I}{\mu\text{A}}$	$\frac{I}{\mu\text{A}}$	$\frac{I}{\mu\text{A}}$	$\frac{I}{\mu\text{A}}$
-90	0.2	0.0	29.0	39.4	31.2
-80	4.0	3.2	24.2	39.4	37.7
-70	11.9	11.2	20.8	39.7	44.2
-60	24.0	21.7	19.5	39.7	49.7
-50	37.1	35.2	20.7	39.7	54.4
-40	51.2	48.9	24.2	39.4	57.4
-30	64.5	61.2	29.2	39.2	58.0
-20	74.2	70.9	35.4	38.5	56.2
-10	80.5	76.7	41.7	38.2	52.0
0	83.2	78.2	47.8	37.7	46.4
10	81.4	75.7	53.2	37.2	39.7
20	74.0	68.5	56.2	37.2	33.3
30	62.5	57.4	57.2	37.2	27.1
40	49.2	43.9	56.2	37.7	22.4
50	34.2	33.2	52.7	38.2	19.6
60	21.2	18.4	47.4	38.2	19.2
70	9.7	8.4	41.2	38.4	20.9
80	3.0	1.7	34.7	39.0	25.1
90	0.7	0.0	28.7	39.2	30.5

b) Half wave plate

Table. 2: Measured current as a function of the analyzer position α for different half wave plate positions φ .

position φ	30°	45°	60°
$\frac{\alpha}{\text{deg}}$	$\frac{I}{\mu\text{A}}$	$\frac{I}{\mu\text{A}}$	$\frac{I}{\mu\text{A}}$
-90	0.0	71.9	56.1
-80	3.5	69.1	43.9
-70	11.7	63	31.7
-60	23.4	52.9	19.3
-50	35.6	42.1	9.5
-40	48.6	29.4	2.4
-30	60.3	17.7	0
-20	69.6	8.0	2.0
-10	74.5	1.9	8.4
0	75.0	0.0	18.8
10	72.0	2.8	30.8
20	64.6	9.9	43.2
30	53.5	19.4	55.3
40	40.8	31.4	64.4
50	28.6	43.9	70.9
60	16.5	54.9	72.8
70	6.8	63.8	70.6
80	1.3	69.7	64.6
90	0.2	71.3	54.8

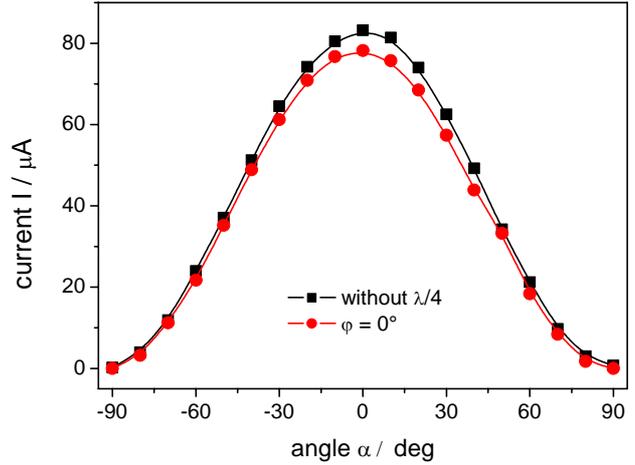


Fig. 3: Current I as a function of the analyzer position α : without quarter wave plate (black) and with quarter wave plate at position $\varphi = 0^\circ$ (red). The solid lines are guides to the eyes only.

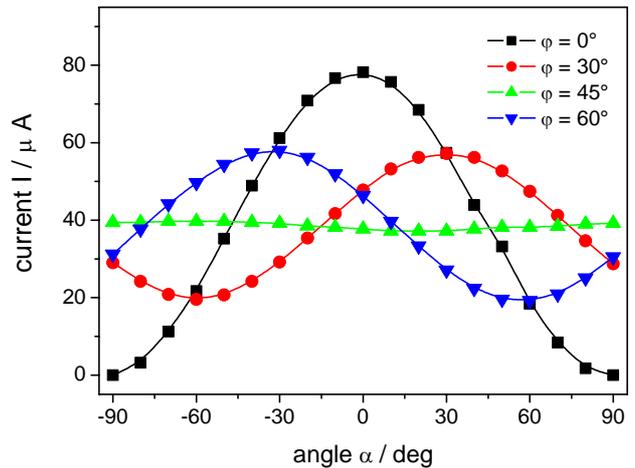


Fig. 4: Current I as a function of the analyzer position α for various quarter wave plate positions φ . The solid lines are guides to the eyes only.

Evaluation and results

a) Quarter wave plate

E_0 is the amplitude of the electric field vector emerging from the polarizer and φ the angle between the polarizer and the quarter wave plate. At a time t the state of vibration of the two component rays is described by:

$$E_1(t) = E_0(t) \cdot \sin\varphi \cdot \sin\omega \cdot t$$

$$E_2(t) = E_0(t) \cdot \cos\varphi \cdot \sin\omega \cdot t$$

In the case of the double refracting quarter wave plates the thickness causes a path difference of $\lambda/4$ (i.e. a phase difference of $\pi/2$) between the two rays. When emerging the quarter wave plate they combine to a resultant ray which can be described by the parametric equations:

$$E_1(t) = E_0 \cdot \sin\varphi \cdot \sin\omega \cdot t$$

$$E_2(t) = E_0 \cdot \cos\varphi \cdot \sin\omega \cdot t$$

These equations describe a rotating E vector in the direction of propagation, i.e. perpendicular to the x- and y-axis about a fixed axis (Fig. 1).

For the angles $\varphi = 0^\circ$ and $\varphi = 90^\circ$ we obtain plane polarized light intensity after the quarter wave plate:

$$I = I_0 \sim E_0^2$$

This is in agreement with the experimental results shown in Fig. 3, i.e. $I \sim \cos^2 \alpha$.

For an angle $\varphi = 45^\circ$ $\sin \varphi = \cos \varphi = \frac{1}{\sqrt{2}}$ and the amount of the rotating E vector is given by:

$$E = \sqrt{E_1^2 + E_2^2} = \frac{E_0}{\sqrt{2}}$$

The light is circularly polarized and the intensity is given by:

$$I = \frac{I_0}{2} \sim \frac{E_0^2}{2}$$

The light is transmitted without loss of intensity in all analyzer positions α . This is in agreement with the experimental results shown in Fig. 4.

At all other angles φ other than 0° , 45° and 90° the transmitted light is elliptically polarized. The tip of the E vector rotating about the axis parallel to the direction of propagation describes an ellipse with the semi axes a and b:

$$E_1(t) = E_a \cdot \sin \varphi \cdot \sin \omega \cdot t$$

$$E_2(t) = E_b \cdot \cos \varphi \cdot \sin \omega \cdot t$$

In agreement with the experimental results depicted in Fig. 4 the intensity for any angle φ between analyzer and quarter wave plate (here e.g. $\varphi = 30^\circ$ and 60°) is given by:

$$I \sim E_0^2 \cos^2 \varphi \cos^2 \alpha + E_0^2 \sin^2 \varphi \sin^2 \alpha$$

b) Half wave plate

The experimental result for the half wave plate (or two quarter wave plates with same orientation) is summarized in Fig. 5.

The half wave plate produces plane polarized light. For different positions φ of the half wave plate only the polarization plane changes. For example, if the position of the half wave plate is changed about 45° the polarization plane changes about 90° .

The maximum and minimum values are not changed. This is in contrast to the experimental results of the quarter wave plate.

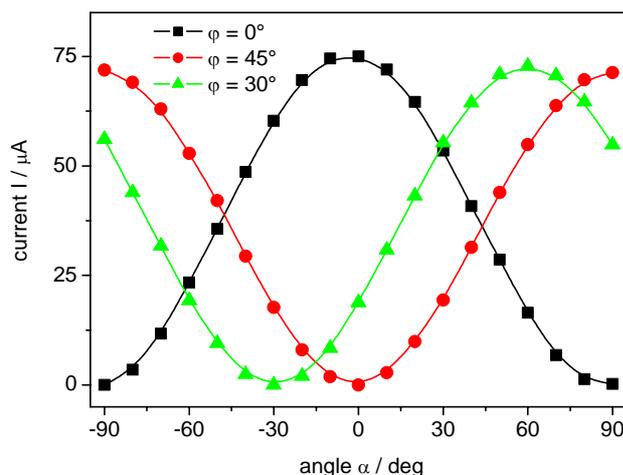


Fig. 5: Current I as a function of the analyzer position α for various half wave plate positions φ . The solid lines are guides to the eyes only.

Supplementary information

Because of dispersion a wave plate will impart a phase difference that depends on the wavelength of the light. Wave plates are thus manufactured to work for a particular range of wavelengths. For the wave plates used here the phase difference produced is best for yellow light. Due to a moderate dispersion in the visible spectrum the deviations are slight.

Wave plates will give the intended effect only when the light penetrates perpendicularly. A slight convergence of the light ray does not affect the experimental results.

Further information about using the polarizer and quarter wave plates can be found in the instruction sheet 472 60.

