Determining the adiabatic exponent c_P/c_V of various gases using the gas elastic resonance apparatus

Experiment Objectives

- Determine the resonance frequency of the system.
- Calculate the adiabatic exponent of the gas used.
- Use different gases.

Principles

In a precision tube with scales for measuring volume, a gas column with the cross-section A, whose volume V, pressure p and type of gas can be varied, is brought to resonant vibration. A magnetic piston inside a glass tube is moved through an electromagnetic alternating current field as a suspended mass m. This periodically compresses and expands the gas (adiabatic constitutional change).

If the frequency of the alternating current field is equal to the resonance frequency of the system the oscillation amplitude of the piston will be maximized (the minimum off-resonance due to friction can be neglected). The time of vibration T for the glass tube closed on both sides and the piston in the middle is:

$$T = \frac{2\pi}{A} \cdot \sqrt{\frac{m \cdot V}{2p\kappa}} \qquad (1)$$

With V = l * A and $\tau = 2\pi$ f it follows that k:

$$\kappa = \frac{2\pi^2 \cdot l \cdot m}{A} \cdot \frac{f_0^2}{p} \qquad (2)$$

l = Length of the gas column (0.28 m), see Fig. 1
A = Cross-section of the gas column ($1.52 \cdot 10^{-4}$ m²)
m = Mass of the piston ($8.8 \cdot 10^{-3}$ kg)
p = gas pressure
= Air pressure (in the example $1.027 \cdot 10^{5}$ Pa)
f_0 = resonance frequency

If the inherent values due to the apparatus design are used, we obtain

$$\kappa \approx 297{,}1 \cdot s^2 \cdot Pa \cdot \frac{f_0^2}{p} \qquad (3)$$

(Pa = Pascal; s = seconds)

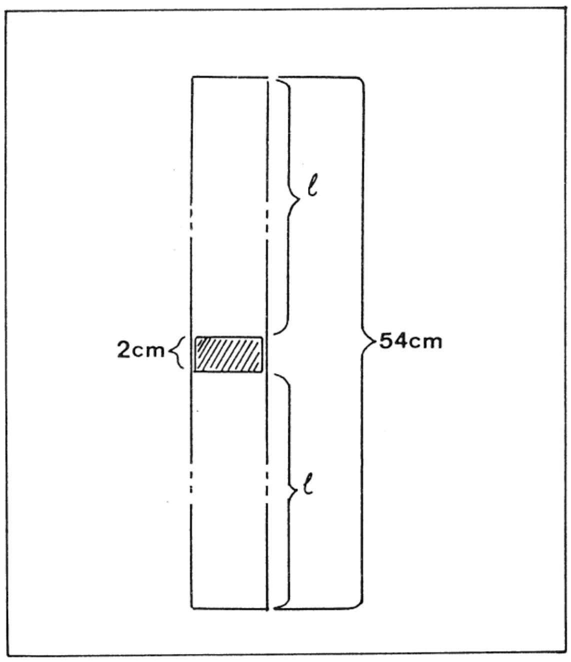

Fig 1: Glass flask of the gas elastic resonance apparatus

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Apparatus

1 Gas elastic resonance apparatus 371 07
1 Digital Multimeter 3340................................531 183
1 Function generator P..................................522 561
1 Stand base V-shape, small 300 02
1 Fine regulating valve 660 980
1 Minican Neon 660 985
1 Minican Carbon dioxide...............................660 999
1 3-way-valve 665 255
1 Silicone tubing...667 194
1 Rubber tubing..604 481
1 Connector..604 510
1 Connecting lead 50 cm, blue.......................500 422
1 Connecting lead 100 cm, red/blue, pair.......501 46

Derivation of equation (2)

The force equation applies to the tube opened on one side:

$$m\ddot{x} = -dp \cdot A \qquad (4)$$

Because this vibration is an adiabatic process, the equation $p \cdot V^{\kappa}$ = constant (Poison's law) applies.

It follows that:

$\frac{dp}{dV} = -p \cdot \frac{\kappa}{V}$ and with (4)

$$m\ddot{x} = -p \cdot \frac{\kappa}{V} \cdot dV \cdot A$$

With $dV = A \cdot x$ and $\frac{V}{A} = l$ and the linear force $F = -D \cdot x$ we obtain the following:

$$m\ddot{x} = -\frac{p \cdot \kappa \cdot A}{l} \cdot x = -D \cdot x. \qquad (5)$$

For the glass tube enclosed on both sides and the piston in the middle, the direction force D is twice as large:

$$m\ddot{x} = -\frac{2 \cdot p \cdot \kappa \cdot A}{l} x = -D^* \cdot x \qquad (6)$$

From the relation for the time of vibration of a harmonic vibration:

$$T = 2\pi \cdot \sqrt{\frac{m}{D^*}}$$

and the relationship (6) for the direction force D* we obtain the frequency:

$$f_0 = \frac{1}{2\pi}\sqrt{\frac{D^*}{m}} = \frac{1}{2\pi}\sqrt{\frac{2p \cdot \kappa \cdot A}{l \cdot m}} \qquad (7)$$

(2) follows from this.

Setup

Before inserting the gas resonance apparatus into the experiment arrangement, turn the tube with opened valve by 180° in the opposite direction of the operating position, so that the piston can slide to the upper end of the tube; close the valve immediately and set up the experiment as shown in Fig 2. For instructions on how to fill the tube with different gasses see the end of this leaflet.

Let the piston (representing the end of the gas columns) slide in the center position needed for the experiment by briefly opening the valves.

Attach the coil so that the upper edge is at the same level as the lower edge of the piston (Fig. 2).

Important

The inside of the glass tubing must definitely be free from any type of dirt, so that the piston can move freely. It can be cleaned with a bottle cleaner moistened with alcohol.

Definitely avoid touching the surface of the piston with bare skin.

Notes

If the glass tubing is very clean, then it can also be used in a horizontal arrangement. This has the advantage that the piston remains in one position and does not sink slowly to the bottom, thereby making it more difficult to read the measured values.

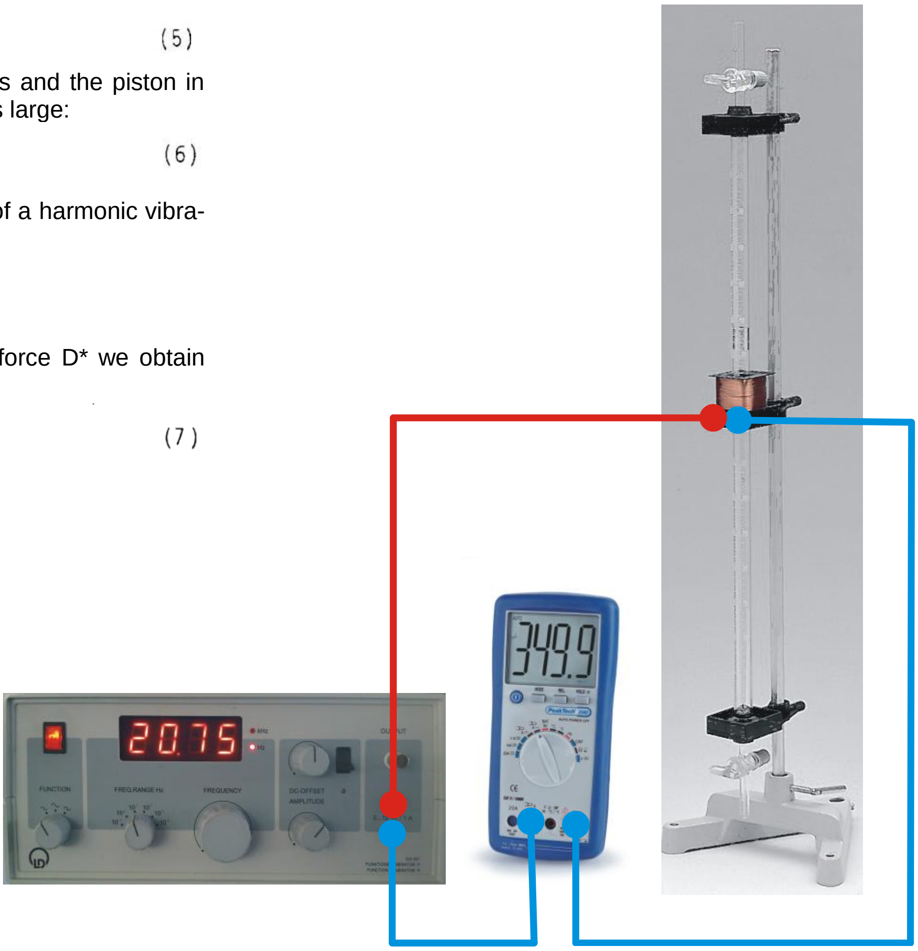

Fig 2: Setup of the experiment

Carrying out the Experiment

Safety note

Caution: Never let any substantial pressure build up in the glass during filling with other gases, the glass tube might explode

First choose a frequency of approx. 50 Hz and set the AC current to 1 A using the amplitude adjustment of the function generator (ordinary multimeters show correct values only for 50/60 Hz). Take a note of the position the amplitude setting and do not exceed it during further experimentation for longer times. Watch the overload LED of the function generator.

To find the resonance frequency, slowly increase the generator frequency from 20 Hz onwards, until the maximum amplitude of the oscillating piston just passed; then lower the frequency until the maximum amplitude as seen before is achieved again. After each setting of a new frequency, the piston will need some time to reach the final amplitude.

If the friction inside the tube is very low, the amplitude of the piston in resonance might be to large. Reduce the AC current amplitude appropriately.

Adjusting the last digit of the function generators frequency will require a calm hand. It is very hard to adjust the last digit stepwise, sometimes it is easier to do a jump forward and backward to set the desired frequency.

For infinite vertical operation without the piston sagging, it is necessary to add a small DC current to the coil using the DC offset switch of the function generator, but the additional force will also influence the oscillation frequency of the piston and therefore the measured values. Therefore this is more for demonstration than for precise student's measurement.

Measurement Example

Using the constant dimensions of the apparatus we derived the formula

$$\kappa = 297{,}1\frac{f_0^2}{p} = \frac{297{,}1 \cdot Pa \cdot s^2}{1{,}027 \cdot 10^5 Pa} f_0^2$$

$$\kappa = 2{,}893 \cdot 10^{-3} s^2 \cdot f_0^2$$

And for example we measured:

Resonance frequency of air: f_0 = 21.9 Hz

Resonance frequency of CO_2: f_0 = 21.0 Hz

Resonance frequency of Ne: f_0 = 23.0 Hz

Evaluation

From the measuring example, we can calculate:

$$\kappa_{air} = 2.893 \cdot 10^{-3} s^2 \cdot (21.9)^2 Hz^2 = 1.39$$

$$\kappa_{CO2} = 2.893 \cdot 10^{-3} s^2 \cdot (21.0)^2 Hz^2 = 1.27$$

$$\kappa_{Ne} = 2.893 \cdot 10^{-3} s^2 \cdot (23.0)^2 Hz^2 = 1.53$$

Theoretical values

According to kinetic gas theory, the adiabatic exponent is (f+2)/f,

With f being the degrees of freedom inside the gas molecule.

$\kappa_{air} = 1.40 = \frac{7}{5}$; diatomic, f=5, 3 translation + 2 rotation

$\kappa_{CO2} = 1.29 = \frac{9}{7}$; linear triatomic, f=7

$\kappa_{Ne} = 1.67 = \frac{5}{3}$, monoatomic f=3, 3 translation

How to perform the experiment with other gas fillings

After finishing the setup (Fig. 1) place the gas apparatus horizontally on the table (Take the whole apparatus out of the stand base).

Step 1: Open all valves and let the piston slide to the end shown in the Fig. 1. To do see whether the piston slides freely to the end tilt the apparatus.

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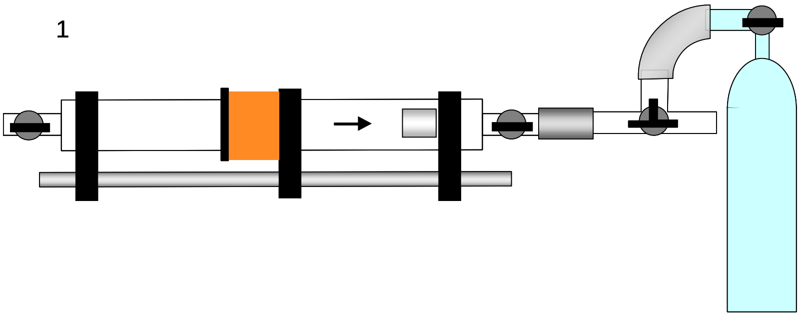

Step 2: Close the open end of the T-valve between the Minican and the gas resonance apparatus.

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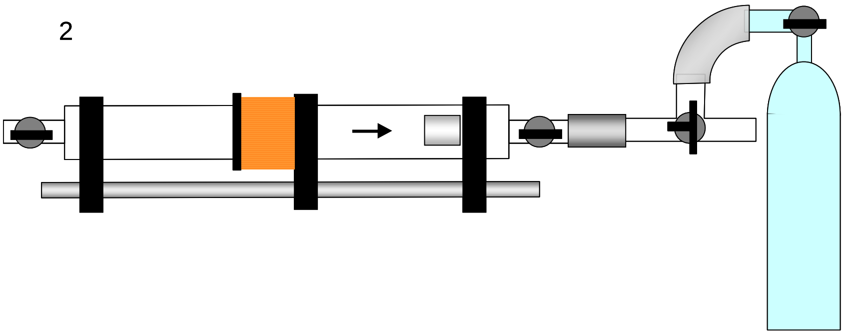

Step 3a: Open **very carefully** the valve of the Minican! Assuming that the check of sliding was successful (step 1) the piston will start to move towards the other end of the gas resonance apparatus. The speed of the piston can be controlled easily by valve of the Minican. Open the valve only to that extend (reduced gas stream) to establish a slow speed of the piston!

Thus the air inside the glass tube is pushed out.

Note: If the piston gets stuck you may knock gently at the glass tube to achieve a further movement.

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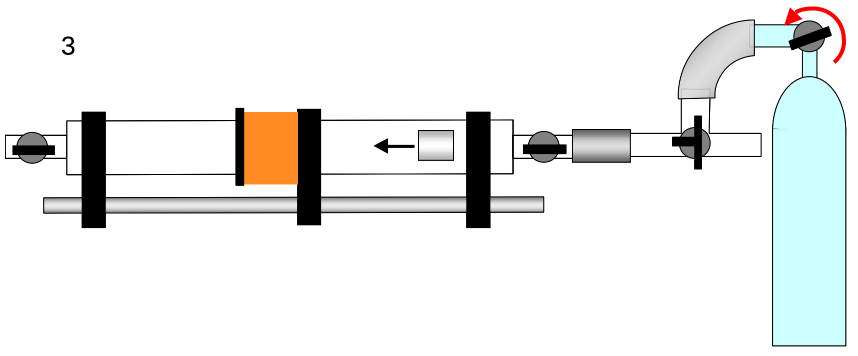
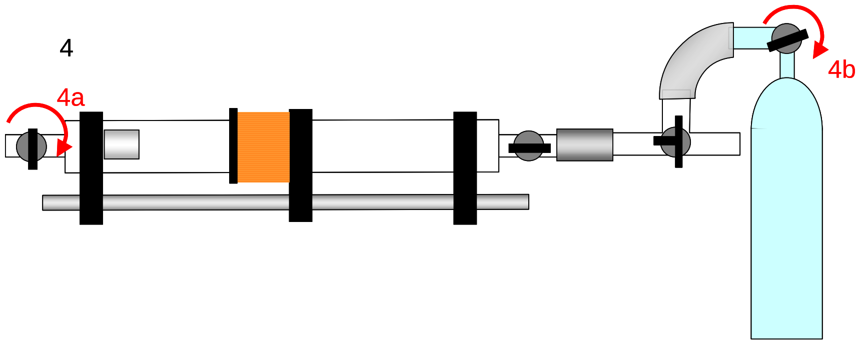
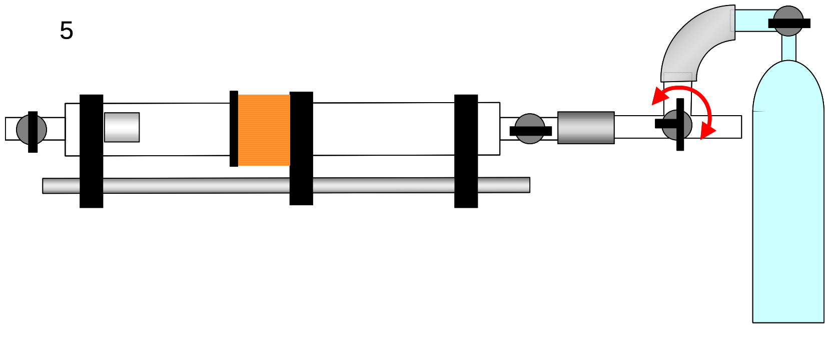


Step 3b: Decrease the gas pressure, i.e. close the valve of the Minican slightly when the piston has reached the other end of the glass tube.

Step 4: After you have reduced the gas pressure in step 4 close first the end of the glass tube with the piston (4a) and then immediately the valve of the Minican (4b).



Step 5: Open shortly the T-valve to expand the gas to normal pressure. Thus the gas pressure becomes the same as the air pressure.



Step 6: Close the valve of the gas resonance apparatus opposite of the piston.

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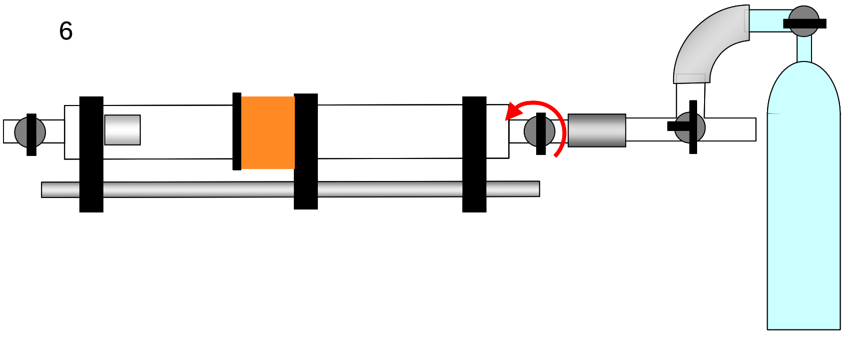
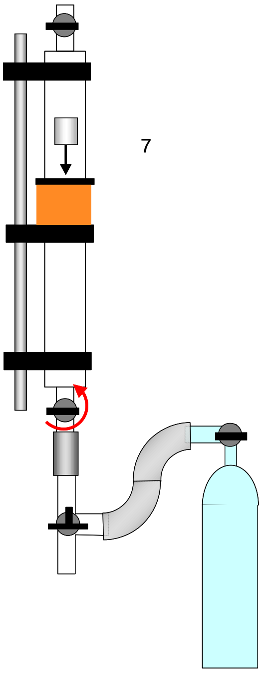


Step 7: Position the gas resonance apparatus vertically to perform the experiment.

The piston will gradually move towards the middle of the glass tube. While this movement downwards the resonance frequency can be adjusted and measured as a test. To do so you have to move the coil towards the piston. The most accurate measurement is done when the piston is approximately in the middle of the tube.



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